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A QUANTITATIVE STUDY OF THE PRODUCTIVITY OF THE FORAMINIFERA IN THE SEA

EARL H. MYERS

Woods Hole Oceanographic Institution, Contribution Number 314

ABSTRACT

Samples of populations of the foraminifer *Elphidium crispum* Linnaeus numbering from 500 to 1,000 individuals were taken both in the littoral and sublittoral zones of Plymouth Sound, Plymouth, England, at intervals of one month for 12 consecutive months. Percentage frequency distribution curves based on the number of chambers in the tests as determined from protoplasmic casts revealed that the life span in tide pools is usually one year, and the life cycle including a sexual and an asexual phase two years, while below low tide level two, three or even four years are required. Growth is limited to the spring and summer months and the rate of growth is 40 per cent greater and the diameter of the test 60 per cent larger in the sublittoral zone. Sexual and asexual reproduction as determined from the presence of juvenile individuals in the samples and cytological evidence of gametogenesis are limited to March and April, although some asexual reproduction takes place in the sublittoral zone in September. From the number of megalospheric juveniles produced in cultures and the ratio of microspheric to megalospheric individuals in the sea it was determined that the annual rate of increase is about thirtyfold and fortyfold in the littoral and sublittoral zones respectively. Populations well in excess of 1,000 individuals per sq. ft., were observed at a number of stations. In a later paper an attempt will be made to correlate these observations on the sequence of events in the life activities of this species with measurable ecological conditions in the sea.

INTRODUCTION

AN inspection of geological maps of the continents will reveal that rocks of marine origin cover approximately 70 per cent of the land areas. One of the principal biotic constituents of these rocks as well as that of recent marine sediments are the calcareous tests of foraminifera. Because of their universal distribution in ancient seas, the relative abundance, and the persistent nature of the calcareous tests, they provide a most useful guide in making stratigraphic correlations. In any attempt to comprehend the biological significance of the foraminifera in recent and geologic marine sediments we must consider the probable rate at which these tests have accumulated on the floor of the sea. This problem immediately calls forth such questions as: How numerous are living foraminifera in the sea? At what rate do they reproduce? What is the life span? What factors influence the rate of growth? and What proportion of the tests are destroyed by organisms living in or above the sediments? Although there is a vast and growing literature on the foraminifera it contains virtually nothing that would contribute to a comprehensive solution of any of these problems (Cushman 1940).

Elphidium crispum Linneus (*Polystomella crispa*), the species made classic through the work of Lister (1895) and Schaudinn (1903) on the life cycle is considered to exemplify best, this group of marine Rhizopods. Further geographical subspecies of *E. crispum* are to be found in all coastal regions, therefore this seemed to be the logical material for this first attempt to provide answers, based on actual observations, to the problems suggested above.

When foraminifera are killed, fixed, decalcified, stained and cleared, it is possible to note the size, arrangement, and total number of chambers. When chamber counts for samples of populations taken monthly for twelve months are plotted as frequency distribution polygons, much may be learned concerning seasonal variations in the rate of growth, reproduction, periods of dormancy, and the varying proportion of microspheric to megalospheric tests. The described methods also provided an abundance of material for cytological study.

For a statistical study of populations a dependable source of material is essential. In Plymouth Sound on the southwest coast of England *E. crispum* is abundant in two life zones. From small tide pools it was possible to take representative samples of the populations almost as readily as from a culture dish, and in the sublittoral zone equally representative samples were obtainable with the aid of a dredge from areas in which only slight variations in sediments and bottom configuration were observed. The population per unit area in these regions was determined from samples taken with a special suction grab, the number of living foraminifera in a sample being determined from the number of

protoplasmic casts that remained after the foraminifera had been treated with strong acidified alcohol.

The studies of Lister (1895) on the life cycle of this species were also made at Plymouth, and our observations of the activities of living foraminifera and permanent cytological preparations provide additional evidence in support of his work. Our results also explain in part why reported attempts to culture any but the smallest species of foraminifera have failed, and why the cytological behavior of the nuclei in division are so inadequately known.

Although our studies on *E. crispum* or its geographic subspecies extend over a period of ten years in the Pacific, the Atlantic, the Mediterranean, and the Java Sea, and during this time we have probably prepared for cytological study well in excess of 100,000 megalospheric individuals, many hundreds of which were in some stage of gametogenesis, we have failed to obtain evidence as to the fate of the relatively enormous nucleus that is characteristic of this phase of the life cycle. It is to be hoped that data on periods of reproduction contained in this paper will contribute to the solution of this fundamental biological problem, for in its solution depends to a major degree the ultimate fate of the chromidial theory of Hertwig which is now in a state of disrepute, but definitely has not been eliminated from consideration (Calkins 1933, p. 456, 236, and 87).

Hydrographic Conditions at Plymouth

Plymouth Sound on the southwest coast of England, has an average depth of 30 ft., and an extreme tidal range of 16 ft. Several streams with wide estuaries empty into the Sound and during periods of heavy rainfall these affect the salinity, particularly at the surface where it varies with each tidal phase and from season to season. The normal range of salinity is from 34.9 parts per thousand to 25.3 parts per thousand with an occasional low of 19.5 parts per thousand (Fig. 1). The mean monthly surface water temperatures vary from 16° C. in August to 8° C. in February. The estuaries and the deeper parts of the Sound have mud bottoms while on either side of the main channel the bottom is covered with quartz sand and small shell gravel. The abrupt rocky shores are fringed with dense growths of *Fucas*, *Lamminaria*, and other algae and the numerous tide pools on the reefs usually contain dense growths of *Corallina*.

As is typical in the higher latitudes, there is an abrupt and prolonged spring outburst of phytoplankton which soon depletes the nutrient salt content of the water. This is followed by several minor summer outbursts, and with the regeneration of the nutrient salt content of the water in the fall there is a second major outburst that is less intense and of shorter duration than that of the previous spring. Following the spring outburst of phytoplankton there is a rapid increase in the zooplankton population, and a more gradual increase extending into the fall. The reduced period of sunlight in the late fall limits the production of phytoplankton and there is a rapid reduction in the zooplankton population. The periods of growth and reproduction in the foraminifera are somewhat similar to those of the zooplankton and these life activities in both groups are no doubt related to the food organisms available in the microflora of the sea.

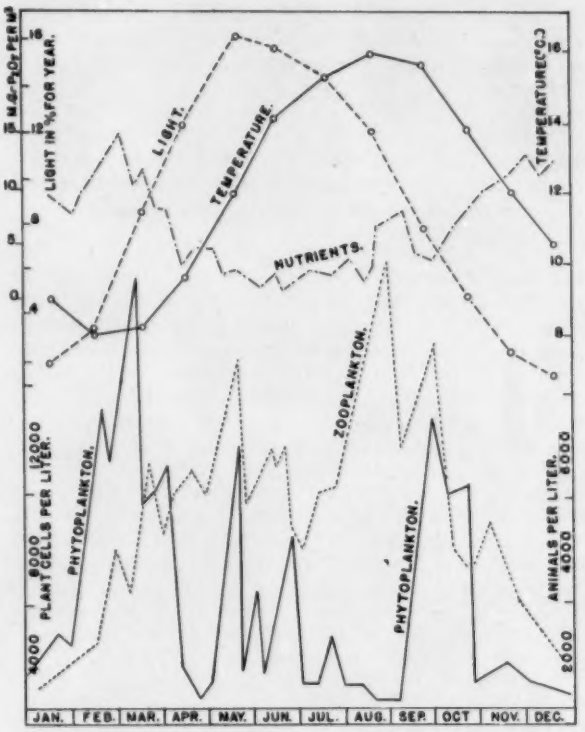

Fig. 1. Graphic presentation of conditions in the sea in the vicinity of Plymouth, England.

Material

From June 1938 to July 1939, a number of stations in the vicinity of Plymouth were worked monthly except at the height of the reproductive season in March and April when additional samples were taken. From several stations within Plymouth Sound enormous numbers of individuals were obtained with the aid of a dredge from a bottom composed of clean sand, shell fragments, and other neritic material, while smaller numbers were taken from tide pools containing a heavy growth of coralline algae both within the Sound and along the adjacent coast. Samples of sediments from the English Channel in the vicinity of Plymouth contained few living individuals of this species. The data for the frequency distribution curves presented in this paper were obtained from material taken in a small tide pool that was little more than a cleft in the rocks at about three-quarter tide level on the southeast side of Drake's Island, and dredgings from the sandy bottom north of the Island at from 5 to 7 meters below low tide level and only a few hundred meters removed from the tide pool station. Series of curves from other geographical regions will be presented in a later paper, and an attempt will be made to explain in part the influence of varying ecological conditions upon test morphology, growth, life span, periods of reproduction, and other life activities.

Methods

Tide Pool Sampling.—*E. crispum* living on coralline algae, growing on the bottom and sides of small tide pools were obtained by removing handfuls of the algae including hold fasts and attached debris from representative areas within the pool. This material was then washed vigorously between the hands in a tubular sack, the bottom of which was a piece of silk netting having 2 mm openings and the top attached to a wooden ring that rested on the rim of a deep container filled with sea water. The sediments including the foraminifera were freed from silt and organic debris by repeated washing and decantation. Even when considerable care was used, some of the smaller foraminifera including two to four chambered *E. crispum* were lost. This loss, however, had only slight influence upon the frequency distribution curves of growth stages within a population, and no effect upon the interpretation.

Most plant and animal organisms living within the intertidal zone are better adapted to withstand several hours of slow desiccation than to remain in stale water for a much shorter period. For this reason when it was more convenient to prepare the material in the laboratory than at the collecting grounds the following procedure was adopted: The coralline algae were placed in a container which permitted the water that drained from the algae to escape, and the whole was then covered by larger algae in order to provide shade, and some cooling through evaporation. If the temperature in the container is not allowed to increase by more than a few degrees above the surface temperature in the sea, the foraminifera may remain for twelve hours or longer in a suitable condition for starting cultures.

Dredged Samples.—Dredgings were made with a one meter D-net, and when skillfully handled, samples consisting largely of surface sediments were taken. Much of the quartz sand which made up the bulk of these samples was dispensed with by placing several liters of the sample in a twelve liter container filled with sea water and then agitating the whole in such a manner as to cause the foraminifera, shell fragments, and other material of low density to collect at the surface of the heavier sand. This surface material including the foraminifera was then removed and washed as described in handling tide pool samples. By careful manipulation it was possible at times to obtain samples a liter of which contained several thousand *E. crispum*. Microscopic examination of the discarded sand usually revealed few foraminifera, and again it was of little consequence that a few of the earlier developmental stages were lost.

Unit Area Samples.—Population counts were made from samples of sediments taken with a Hunt Suction Grab Sampler (Hunt, 1926–27). This grab consists of a compression chamber provided with a delivery tube that empties into the upper portion of the chamber, and is closed at the lower end by a glass disc. The glass disc is held in place by heavy grease and a tubular brass sleeve provided with ports in which the smaller end of a funnel-shaped sampling device slides freely in such a manner that the ports are closed when the grab strikes the bottom, and at the same moment a plunger smashes the glass disc. The air in the chamber is compressed by the inrush of a watery sample of bottom sediments driven through the delivery tube into the

chamber at a pressure that is equal to the weight of the column of water above the bottom. As the grab is hauled to the surface the sample is not subjected to washing, nor can it otherwise escape as so often happens with many sampling devices when there is an appreciable amount of shell fragments and other coarse material in the sediments. The surface sample of sediment taken is three inches in diameter and when the grab is used in deeper water the volume of the sample may be controlled by placing an amount of water in the chamber that is equal to the difference between the volume of the sample desired and the volume of water that would enter the chamber at a particular depth. After the sample is removed it is washed in a tubular sack made of No. 25 bolting silk that retains the smallest foraminifer, and at the same time allows the escape of clay and silt particles. From samples so taken it is possible to estimate the population per unit area on the floor of the sea by determining the number of living foraminifera in the sample after treating it with a 10 per cent solution of formalin at 65° C., and then counting the protoplasmic casts that remain after the calcium carbonate of the tests has been removed with acidified alcohol.

Cytological.—Paraffin sections and total mounts were made from material fixed in Schaudinn's fluid at 65° C., although equally satisfactory results were obtained by first killing in hot water at 65° C., and then adding sufficient formalin to make a 10 per cent solution. The foraminifera were separated from other sediments in 70 per cent alcohol with the aid of a low power binocular microscope using 4× oculars and 8× objectives. Much time was saved by using a sorting chamber made from a 10 cm petri dish, the bottom of which had been marked with a diamond pencil into parallel lines so that the distance between two lines was a little less than the diameter of the field of the microscope. Only sufficient sediment was placed in the dish to cover the bottom with scattered particles, so that few foraminifera would be covered with other objects. By using the parallel lines as guides it was unnecessary to inspect the same areas a second time except in a superficial manner. The foraminifera were removed from the sorting chamber with a mouth controlled pipette and were transferred to a 5 cm syracuse watch glass. Decalcification of the tests was accomplished in 85 per cent alcohol to which sufficient hydrochloric acid had been added to cause bubbles of carbon dioxide to form slowly on the walls of the test, then immediately adding neutral alcohol to stop all effervescence. This step should be carried out in a comparatively large volume of alcohol and should not be completed in less than twelve hours or there is danger of destroying the relationship of the chambers and the canal system.

Ehrlich's Acid Haematoxylin proved a suitable stain for total mounts, although when material was to be prepared as thin sections they were stained in alcoholic Eosin. The latter stain has a strong affinity for the plastin in the nuclei of foraminifera and should be differentiated in acidified alcohol in the same manner as when using Ehrlich's Haematoxylin. After clearing it is possible to select those individuals which show evidence of reproductive activity or other characteristics that make it desirable that they be sectioned. Heidenhain's Iron Haematoxylin was used on sections.

Foraminifera used for population studies were decalcified, stained, and cleared in clove oil and the number of chambers in each individual determined without removing the foraminifera from the original container. This is necessary since several of the distal chambers are often devoid of cytoplasm and the delicate chitinous lining of these may be easily lost.

The Life Cycle of *Elphidium crispum*

In the life cycle of *Elphidium crispum* as proposed by Lister (1895) there is a cyclical alternation of sexual and asexual generations. This alternation of generations results in two types of individuals which cannot be recognized from external characters except in the earliest stages, but the morphological beginning and cytological organization of both are distinguishable at all times, see Plate I.

In the sexual phase, biflagellated gametes associate in pairs in fertilization and the resulting zygotic amoebula becomes invested in a calcareous test. Subspherical and later crescent-shaped chambers are added to this initial chamber one at a time in ever increasing size to form a bilaterally symmetrical involute spire. Immediately following fertilization there is a rapid increase in the number of nuclei so that the sexually produced agamont generation is multinucleate from the earliest observed stage.

Prior to asexual reproduction a second period of nuclear division takes place. The protoplasm and contained nuclei then escape from the test

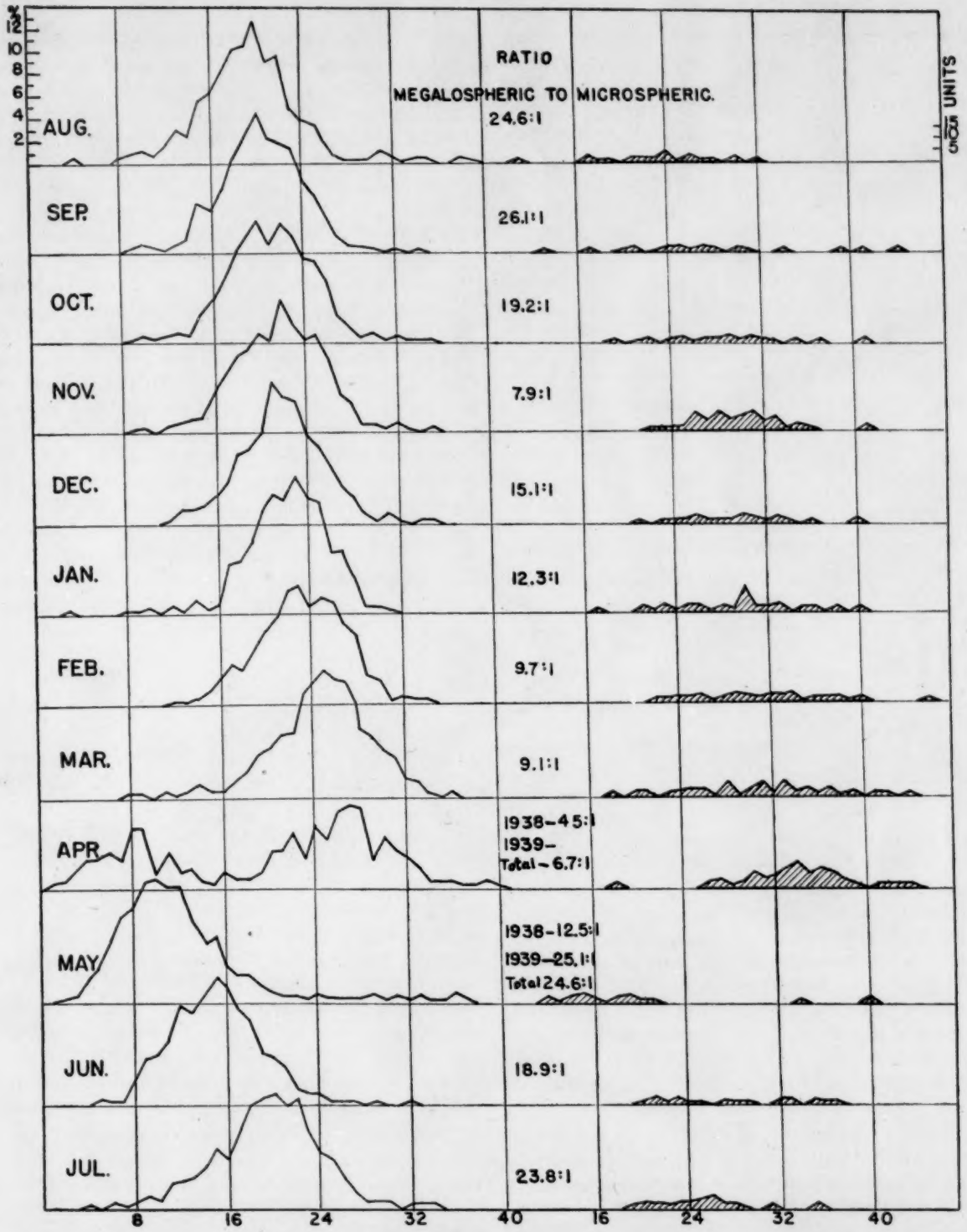

FIG. 2. Frequency distribution curves of growth stages in terms of the number of chambers in 500 *E. crispum* taken at monthly intervals for 12 months from a tide pool on Drake's Island. Curves for the megalospheric agamonts are based on percentage frequency distribution for each class, while the actual number of microspheric agamonts in the sample is represented. The changing ratio of microspheric to megalospheric individuals in April and May followed by the occurrence of juveniles of the respective generations show that the period of reproduction is brief, that the life span is one year and that two years are required to complete the life cycle.

and multiple fission of the somatella results in as many ameobulae as there were nuclei present. The single nucleus of these asexually produced individuals does not divide during vegetative life. Since these amoebulae have a diameter in excess of five times that of the zygotic ameobulae, there is a corresponding difference in the relative size of the initial chamber or proloculus of the tests of the two generations and these are referred to as the megalosphere and microsphere respectively, and with the addition of chambers megalospheric and microspheric tests. In megalospheric tests only the proloculus is spherical all other chambers being crescentic in shape.

In the mononucleate megalospheric gamont, gametogenesis begins with a rapid increase in the numbers of nuclei with a corresponding diminution in size. Multiple fission of the somatella in this generation takes place within the test and results in mononucleate gametocytes. A final nuclear division which is immediately followed by binary fission of the gametocyte results in subspherical gametes which become biflagellated. These gametes soon escape from the tests and become free and pelagic. It is probable that fertilization takes place between gametes derived from different individuals soon after their release, as was described by Schaudinn (1903). In both phases of the life cycle reproduction terminates the existence of the individual.

The Life Span

When the number of chambers in each of 500 megalospheric *E. crispum* taken at monthly intervals from a tide pool were plotted for each month as frequency distribution polygons, the resulting curves were nearly normal except for the month of April when a bimodal curve indicative of a second megalospheric generation was obtained (Fig. 2). The virtual elimination of what had been the major portion of the population in March, together with the appearance of enormous numbers of juveniles in May made it evident that the life span of this phase of the life cycle is one year. A similar series of curves for the microspheric generation made it equally evident that the life span of this second phase of the life cycle is also one year, and that two years are required to complete the life cycle.

A series of samples of one thousand individuals from the sublittoral zone were used in a similar study (Fig. 3). These larger samples were desirable because of the wider limits in the numbers of chambers, *e.g.*, 80 as a maximum compared with 40, and the smaller ratio of microspheric to megalospheric individuals, or 1 to 38 compared with 1 to 25 in tide pools. In this second group of curves a bimodal condition exists for most months, both for microspheric and megalospheric individuals, and it is evident that the two major modes represent different generations. The maxima extremes for the numbers of chambers in individuals from this life zone do not change appreciably throughout the year, but in the series of curves for the megalospheric generation there are two minima modes, the major coming in May and the minor in October. The juveniles produced in the fall failed to survive the winter so that the only influence on the population was to decrease the ratio of microspheric to megalospheric individuals the following spring. No cytological or other evidence was observed that would suggest a corresponding period of sexual reproduction in the fall. As was stated, the curves for May through August are bimodal, but in September the more rapid growth of the juveniles compared with the rate of growth in those surviving from the previous generation resulted in the overlapping of the two curves, so that the curves for October to May had but a single mode. A bimodal curve was again obtained in May, and it is evident that the first mode represents individuals produced in the spring, while the second represents those that had failed to reproduce, and would therefore be two years old the following spring. Microspheric individuals were so in

FIG. 3. Frequency distribution curves of growth stages in terms of the number of chambers in 1,000 living *E. crispum* taken with a dredge at 5 to 7 meters north of Drake's Island. The smaller monthly samples obtained during the period of winter dormancy were treated as a composite sample. The occurrence of vast numbers of megalospheric juveniles in May and a smaller number in October show that there are two periods of asexual reproduction, while cytological preparations made it evident that sexual reproduction takes place in March and April only. The bimodal curves for the megalospheric phase in May, June, and August represent at least two generations that were produced in different years and suggest that larger individuals are at least two years old. The same is true of the microspheric generation so it is evident that the life span for the respective generations is at least one year, and that two, three or four years are required to complete the life cycle in this life zone. For comparison curves for the tide pool populations from the adjacent foreshore are represented in broken lines. Stippled area in the curve for March represents that part of the population showing some phase in gametogenesis.

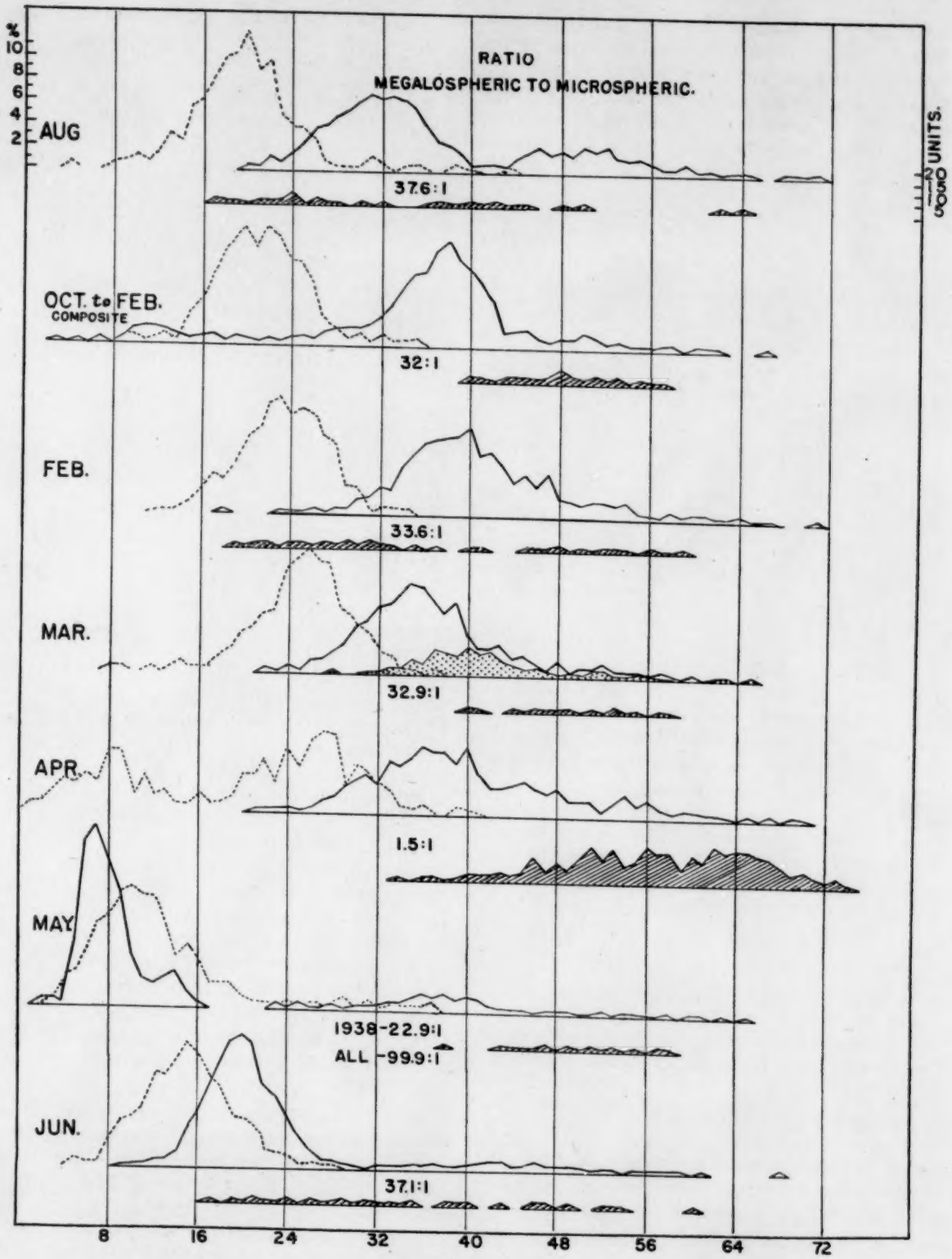
RATIO
MEGALOSPHERIC TO MICROSPHERIC.
AUG
37.6:1
OCT. to FEB.
COMPOSITE
32:1
FEB.
33.6:1
MAR.
32.9:1
APR.
1.5:1
MAY
1938-22.9:1
ALL -99.9:1
JUN.
37.1:1
2 UNITS.
8
16
24
32
40
48
56
64
72

FIG. 3.

the minority and the limits in numbers of chambers such, that the curves for this phase of the life cycle are somewhat more difficult to analyze. One of the most significant features of the series is the enormous increase in the ratio of microspheric to megalospheric individuals in April, and the return to the more normal ratio one month later. Since we know that reproduction terminates the existence of the individual this abrupt change can have but one explanation and that is that sexual reproduction in the early spring had resulted in the elimination of most of the megalospheric gamonts that were then either one or two years old, and that later asexual reproduction by a similar proportion of one and two year old microspheric agamonts resulted in the restoration of approximately the former ratio.

Since the life span of megalospheric *E. crispum* living in the sublittoral zone is either one or two years, and the maxima extremes in numbers of chambers are similar in both generations, it follows that the life span of microspheric individuals is also one or two years, and the life cycle including a sexual and an asexual phase requires two, three or even four years.

In the sublittoral zone, sexual reproduction by megalospheric gamonts precedes asexual reproduction by microspheric agamonts by approximately one month, therefore microspheric individuals are produced one month earlier in the spring and survive one month longer the following spring and therefore have a life span that is approximately two months longer than that of megalospheric individuals. This explains in part the larger size of microspheric tests found in the sediments. In the tide pool the difference in the life span in the respective generations is only about one-half that observed in the sublittoral zone or from three to four weeks.

The life span of *E. crispum* is perhaps longer than that of most foraminifera living in the same life zones. In our experience with cultures we found that under the most favorable conditions *Discorbis globularis* (d'Orbigny) produced a test having nineteen chambers in fifty-two days. Microspheric *Patellina corrugata* Williamson, which is the form usually recovered from the sediments, matures in twenty-one days, while microspheric *Spirillina vivipara* Ehrenberg matures in only ten days, but in the sea, the nature and size of *S. vivipara* suggests that the usual life span is several times this.

Rate of Growth

Winter is a period of dormancy, more as a result of food shortage than of low temperature. This is suggested by the fact that the period of greatest growth and reproductive activity immediately follows the month of lowest temperatures for the year, *e.g.* February, and the excessive shrinkage of the protoplasm observed in material taken in the late winter and early spring which had been prepared by cytological methods that had not produced shrinkage at any season at La Jolla, California, or throughout the summer and fall at Plymouth, England, (Pl. I, Fig. 8). In March there was a gradual refilling of the chambers beginning with the distal and later extending to the earlier formed chambers. In dredged samples taken north of Drake's Island on February 14, when the water temperatures were minimum for the year, renewed activities of the foraminifera had already resulted in the filling out of the distal three to fourteen chambers, and in 52 individuals in a sample of 1,133, or 4.5 per cent all chambers were filled. One month later on March 14, 386 in 1,155 or 33.4 per cent showed little shrinkage, and the remainder were rapidly filling. This individual difference in the time when activities were resumed depended to some extent upon the time water turbulence disturbing the bottom sediments uncovered the foraminifera that had been buried as a result of turbulence earlier in the season, and how soon they were able to avoid becoming reburied to a depth from which they were unable to escape, a depth that does not exceed 5 to 8 times the diameter of the test, or for large individuals about 1 centimeter.

A sample taken from a tide pool on Drake's Island on March 14, contained only 126 in 500 individuals or 25.2 per cent in which the tests were filled with protoplasm, while the remainder still showed considerable shrinkage. This was 8.2 per cent lower than was observed in a sample taken in the sublittoral zone on the same date. This was unexpected for in the tide pools the foraminifera were free to begin feeding and growth activities as soon as conditions became favorable.

The rate of growth in a population as determined from the mean rate at which chambers are added to the tests is more dependable than linear measurements of the tests, although satisfactory results could be obtained through the use of curves based on measurements of ground

sections of tests (Fig. 4). Chambers added in the early spring are wider, longer, and thicker than those produced during the late spring and summer, and especially the final chamber or two

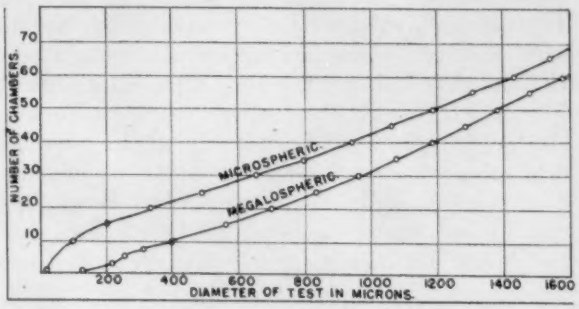

FIG. 4. Curves based on measurements of ground sections of tests of *E. crispum* taken in the sublittoral zone. The mean, maximum diameter of 10 microspheric and 10 megalospheric tests were determined by measuring across the sections at 5 chamber intervals.

produced in the late fall when activities resulting in growth were at a minimum. This usually results in a depression in the outer margin of the test. For these reasons it is possible to determine the number of new chambers added after the resumption of growth in the spring, and in succeeding samples, the rate of growth.

Thus, on March 20, larger individuals from dredged samples averaged 3.2 new chambers, April 29, 5.5, May 12, 9.1, and June 15, 14.7 which was an average of one chamber in 5.9 days. In these large individuals the time required to produce a new chamber, including the secretion of a delicate membranous cyst that partially encloses the test and makes it fast to the substratum, the absorption of water to increase the volume of protoplasm causing the extrusion of a mass of hyaline ectoplasm that gives shape to the developing chamber, and the secretion of the calcareous walls requires from 12 to 18 hours, as was determined from cultures. In the five days remaining, the foraminifera must acquire, digest, and assimilate sufficient material with which to produce the succeeding chamber. At first the rate of growth in tide pool populations was somewhat greater than that observed in populations from the sublittoral zone. This was due in part to the shorter interval between the addition of successive chambers in these smaller individuals.

The data on the rate of growth throughout the late spring and summer are more complete for the megalospheric than for the microspheric generation because of the prevalence of the former in the population, and it is also possible to take large numbers of megalospheric juveniles from the earliest stage whereas it is unusual to take microspheric juveniles before the test consists of 12 chambers because of their smaller size, *e.g.*, the average megalospheric proloculus has a diameter that is about equal to that of the test of a microspheric juvenile having ten chambers, and has several times the mass.

Growth curves based on the mean number of chambers in tests show that by Spetember 15, the average megalospheric individual living in a tide pool had developed 20 chambers and had a diameter of 430 μ, compared with 28 chambers and a diameter of 380 μ for microspheric individuals (Fig. 5). In the sublittoral zone the average megalospheric test on the same date had 33 chambers and a diameter of 1,040 μ and microspheric tests, 47 chambers and a diameter of 1,100 μ. Therefore the rate of growth in numbers of chambers was about 40 per cent more and the diameter of the tests 60 per cent larger in *E. crispum* living in the sublittoral zone

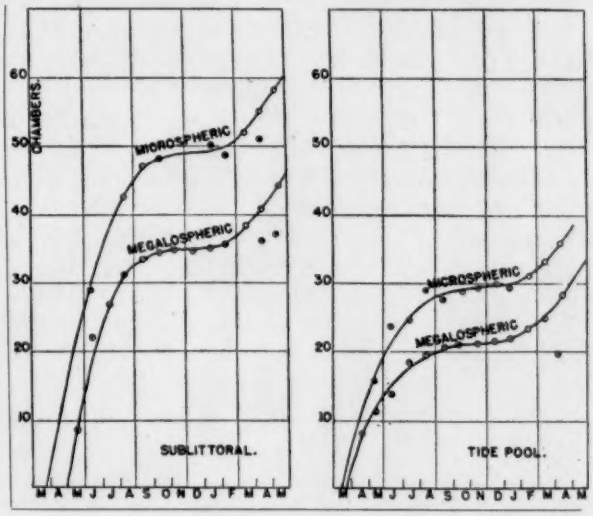

FIG. 5. Growth curves for *E. crispum* obtained by plotting the mean number of chambers in samples taken at monthly intervals. Open circles indicate the mean number of chambers added in the spring of the second year rather than the mean number of chambers for all individuals.

than in those from the littoral zone. In Fig. 6 it will be noticed that the least diameter for megalospheres in individuals taken in the sublittoral zone is larger than the mean diameter for the class of those taken from the tide pools. Similar growth differences have been observed

by us in other species of foraminifera which inhabit these two life zones although there are those, who, with some justification, would regard the two forms as distinct species and it is

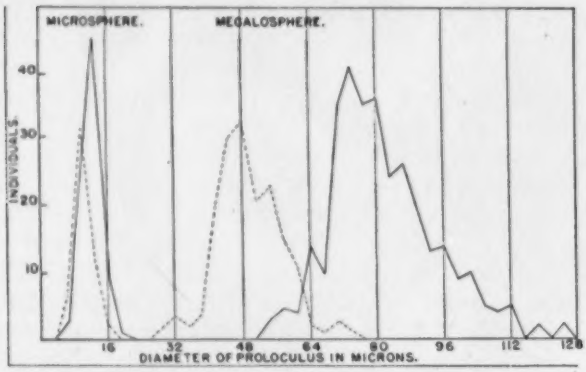

FIG. 6. Frequency distribution curves for the mean diameter of the proloculus of *E. crispum*. Broken lines represent 50 microspheric and 218 megalospheric individuals from tide pools and solid lines 82 microspheric and 352 megalospheric individuals from samples taken in the sublittoral zone.

true that in larger and more complex invertebrates where specific differences are more easily recognized, closely related forms do co-exist.

One example is the common bay mussel *Mytilus edulis* Linnaeus which is most numerous in the intertidal zone, while along the coast of California the larger *Mytilus californianus* Conrad thrives immediately below low tide level. In the case of *E. crispum* Mr. Arthur Earland is agreed that we are probably dealing with a single species and that the observed differences were the result of the conditions under which they were produced.

Data in addition to those presented in Figs. 1 and 2 were used in preparing the growth curves in Fig. 5.

PERIODS OF REPRODUCTION

The approximate dates for the beginning and ending of the reproductive activities can be determined from several sources. These sources include cytological evidence of gametogenesis in the megalospheric generation, Pl. I, Figs. 6 and 7, nuclear division in microspheric individuals which immediately preceeds asexual reproduction, the increasing and diminishing ratios of microspheric to megalospheric individuals in a population (Fig. 7), the number and position of modes in frequency curves for the respective generations (Figs. 2 and 3), and finally the rate of growth in each of the two generations (Fig. 3). From these data we have concluded that in 1939 *E. crispum* living in the littoral zone produced sexual gametes from March 14 to April 18, and that the probability of finding gametes after the latter date was only one in many thousands. Asexual reproduction beginning about March 24 was nearly 60 per cent complete by April 22, although it continued at an ever diminishing rate until July 15, when few mature microspheric individuals remained. It is true that an occasional microspheric individual asexually produces megalospheric juveniles whenever conditions become suitable, but the rare occurrence of megalospheric juveniles having less than 8 chambers after May 15, suggests that it would be most unusual to observe cytological or other evidence of asexual reproduction throughout the summer, and that cultures started at this time

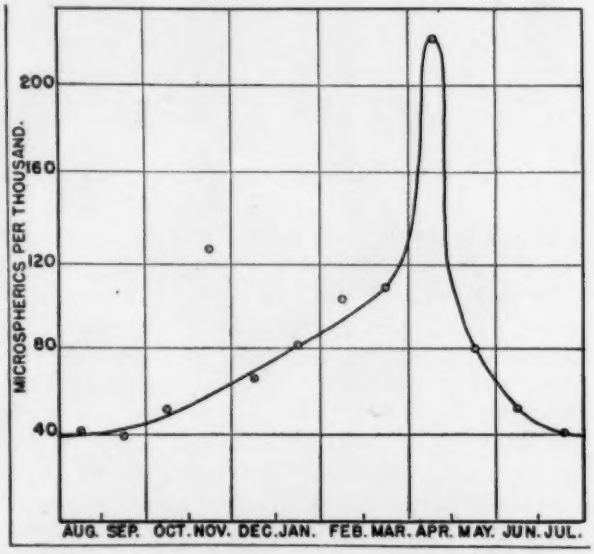

FIG. 7. Seasonal change in numbers of microspheric agamonts per 1,000 individuals in tide pool populations of *E. crispum*. The abrupt increase in the proportion of microspheric to megalospheric individuals in April resulted from a reduction in the number of megalospheric gamonts due to sexual reproduction in March and was followed by an equally abrupt decrease in the proportion of microspheric agamonts in April following asexual reproduction.

of the year would probably fail to produce evidence of either sexual or asexual reproduction for some months.

In the sublittoral zone the period of sexual reproduction extended from March 14, to April

22, and the probability of obtaining cytological evidence of this type of activity after May was again negligible. Asexual reproduction in this life zone was somewhat later and may be associated with the rate of recovery from winter dormancy of the relatively enormous multinucleate microspheric individuals in which the mean number of chambers was 58 compared with 36 for microspheric individuals living in tide pools, although in the sexual phase of the life cycle it was observed that in a population the larger megalospheric individuals reproduced earlier. Judging from the first appearance of the megalospheric juveniles in the sediments of the sublittoral zone asexual reproduction by microspheric individuals began about April 6, and continued until May 30, although an occasional individual may reproduce during the summer. In October megalospheric juveniles were again numerous and from the observed rate of growth and the absence of juveniles in the previous samples it would seem that these had been produced soon after September 15.

Ratio of Microspheric to Megalospheric Individuals

The ratio of microspheric to megalospheric individuals is of interest to the taxonomist because descriptions of both phases of the life cycle are desirable. These descriptions are usually made from ground sections, and it is often necessary to make tens or even scores of preparations to find a microspheric test. The ratio of these two forms within a given species is roughly similar in most regions, but in a population the ratio varies according to the season, and with the substratum upon which they live.

One hand full of coralline algae from a tide pool may show a ratio of microspheric to megalospheric individuals of 1 to 10, and a second hand full from a different part of the same pool, a ratio of 1 to 25. This difference in ratios is probably related to the amount of protection afforded in the respective areas. Throughout the growing season the mean ratio remained 1 to 25, while in the sublittoral zone where there was less protection, the ratio was little more than one-half this, or 1 to 37. In the sediments of the sublittoral zone the ratio of microspheric to megalospheric tests average about 1 to 40, while in the foraminiferal population the ratio from March to September, which is the principal growing season, is about 1 to 37. During a brief interval following the period of asexual reproduction by the megalospheric generation in late April, the ratio was 1 to 2.1, and in one sample 1 to 1. On a mud bottom a ratio of 1 to 99 was obtained as late as June due probably to the failure of the sexual phase of the life cycle in this habitat. In May at the close of the period of asexual reproduction a ratio of about 1 to 37 was resumed in the area north of Drake's Island.

It was stated that in tide pools the ratio during the growing season was about 1 to 25. After the period of sexual reproduction was well advanced, a ratio of 1 to 4.5 was observed, while one month later when asexual reproduction was virtually complete the ratio was 1 to 24.6.

From the above we may conclude that although the ratio of microspheric to megalospheric tests in the sediments remains relatively constant, the ratio in living foraminifera in a population varies within wide limits.

The frequency distribution curves for the mean diameter of the proloculus of microspheric and megalospheric tests from the respective life zones (Fig. 6), are based on measurements of material prepared for cytological study in which some shrinkage had taken place, but since all material was prepared in the same manner any slight error would remain constant for all individuals. From these curves it will be observed that there is a distinct size difference in the two generations as has been generally accepted. This, however, is not the case in all genera, and it is often necessary to use other characteristics such as the shape of the proloculus and the succeeding chambers, the ultimate size and shape of the test, or to note the number of nuclei. In general it may be stated that in most species the small proloculus of the sexually produced microspheric generation is nearly spherical and that many nuclei are to be found in the cytoplasm (Myers, 1940).

Rate of Reproduction

Since reproduction terminates the life of the individual, the rate of reproduction depends upon the number of juveniles produced and the length of the life span. From growth curves we have determined that the life span of *E. crispum* in tide pools is one year. At La Jolla, California, we observed that the number of progeny asexually produced by microspheric individuals removed from tide pools ranged from 52 to 128 with an average for 20 groups of 119, and from the number of nuclei in microspheric agamonts from Plymouth Sound it would seem that this

rate was about equal in the two regions. In sexual reproduction each megalospheric individual produces several thousand gametes. The probability of two gametes uniting in fertilization and the chance of the resulting zygote surviving to a stage where it may be readily recovered from the sediments can be estimated from the ratios of microspheric to megalospheric individuals observed before and after the period of reproduction. These ratios were found to be about 1 to 10 and 1 to 25 respectively. Since microspheric and megalospheric individuals were produced in the ratio of 1 to 25 and the actual number of megalospheric juveniles asexually produced by a microspheric individual was 119, then, individuals of the two generations were actually produced at a ratio of about 4.8 to 119. Prior to reproduction there were ten times as many megalospheric as microspheric individuals, therefore 10 megalospheric individuals produced the equivalent of 4.8 microspheric juveniles, or in other words the gametes from 2.1 megalospheric individuals were required to produce 1 microspheric juvenile that survived to develop a test having ten or more chambers. If 1 microspheric produces 119 megalospheric, and 2.1 megalospheric 1 microspheric, then the annual rate of productivity for the two generations is about fortyfold.

Reproduction was not observed in microspheric individuals taken in the sublittoral zone, but the rate of reproduction may be determined from cytological preparations of individuals about to reproduce, or which were reproducing when killed and fixed. A microspheric individual about to reproduce may be recognized in several ways. Often there is a remnant of a delicate protective cyst clinging to the test composed of animal cement, fragments of diatom frustules and other debris, the protoplasm usually has receded from the inner whorl of chambers and there is evidence of nuclear division or an unusual number of large nuclei of approximately the same diameter, suggesting that most of the nuclei have divided at least once, and that nuclear division was followed by growth of the daughter nuclei. The number of nuclei in microspheric individuals about to reproduce was found to be from 152 to 224 whereas the usual number was 27 to 52. Since the ratio of microspheric to megalospheric individuals in the sublittoral zone before and after the period of reproduction is about 1 to 33 and 1 to 37 respectively, and if we assume that the average number of megalospheric juveniles produced by one microspheric individual is 200, then by the same computation discussed above, we may deduce that 6 megalospheric individuals are required to produce a single microspheric juvenile, and that the rate of increase for the two generations is about thirtyfold, compared with fortyfold for *Elphidium crispum* living in tide pools From the ratios of microspheric to megalospheric individuals in the two life zones it is evident that sexual reproduction is more successful in tide pools where more protection is afforded and circulation is reduced for long periods between tidal phases.

Population per Unit Area

No attempt to distinguish between species of foraminifera which constitute a population from a mere assemblage of tests nor to determine the population of foraminifera on the floor of the sea has come to our attention.

Three samples taken with the Hunt Suction Grab on February 16 north of Drake's Island at a depth of about 7 meters contained 4, 7, and 12 living *E. crispum* respectively, while a similar sample taken on June 15, contained 40, 58, and 62, or approximately five times the number taken in February. Since the grab takes a surface sample 3 inches in diameter, the maximum number of *E. crispum* per unit area observed that had survived the winter was equal to 245 individuals per sq. ft., while the maximum population observed which was in June was 1,225 per sq. ft. This was the greatest population per unit area for this or a comparable species observed by us except in the coral reef areas of the Java Sea. Counts of living *E. crispum* may be easily made because the bright greenish brown color of the protoplasm shows through the walls of the test, while empty tests are either translucent or chalky white depending upon how long the organism has been dead. These counts, however, were later substantiated by decalcifying the foraminifera and determining the number of protoplasmic casts that remained. This step was necessary in making population counts of most species living on mud bottoms where the tests become filled with a precipitate of aluminum silicate, and it is necessary to use biological stains to differentiate living from dead foraminifera.

The determination of the population per unit area in tide pools would have little significance

because of the variable nature of the habitat especially in the small pools that proved so useful for other studies, nor would a comparison of the numbers of foraminifera per unit volume of the sediments from the two life zones contribute appreciably to this problem since the samples were taken by different methods. A comparison, however, is interesting for other reasons. A tide pool sample of washed sediments taken on July 15 contained 91 *E. crispum* per cc, while a similar amount of washed sediment from the sublittoral zone taken on the same day contained 126 individuals of the same species. Tide pool foraminifera remained fairly numerous throughout the winter, although it was necessary to examine much larger amounts of washed sediments to obtain the 500 *E. crispum* desired.

From March to October samples of sediments from the sublittoral zone contained more *E. crispum* per cc than were to be found in half a liter from October to February. In February they were again numerous, and it became evident that the reduced numbers observed during the winter were not due to the death of the foraminifera, nor had they been removed from the area by currents, but had become mixed with the sediments and buried to depths from which they were difficult to recover. Therefore in determining the population per unit area on the floor of the sea from samples taken within the zone of turbulence, it is necessary to consider this mixing of the foraminifera with the subsurface sediments as well as the seasonal variation in the density of populations.

Rate of Survival

Since the life span of *Elphidium crispum* living in tide pools is one year, the annual rate of increase is equal to the rate of mortality, and is in excess of the maximum number of individuals observed in the population at any one time.

We have said that the annual rate of productivity in this life zone is about thirtyfold, therefore the rate of survival from one period of reproduction to the next is about one in thirty. The rate of mortality for juveniles is probably high because they are used as food by myriads of small invertebrates that inhabit approximately the same ecological niche, and surf action no doubt removes large numbers from the pools, most of which fail to reach a suitable habitat. By June the size of the tests make them undesirable as food to most animal associates, and the increasing protection afforded by the growing coralline algae, together with smoother water greatly reduces the rate of mortality during the summer and early fall. October storms which send heavy breakers crashing into the pools erode the algae so that much of the normal habitat of *E. crispum* is destroyed and with it a large portion of the foraminiferal population.

In the sublittoral zone the population near the beginning of the reproductive season on March 14, was 245 per sq. ft. compared with 1,225 per sq. ft. on June 15, which was the maximum population observed. In 1939, 10 per cent of the population, or about 25 per sq. ft., that had survived the winter failed to reproduce so that the 90 per cent which had reproduced gave rise to a population of about 1,200 individuals per sq. ft. which is equal to an average rate of productivity of 5.5 juveniles per individual that had survived till June 15, when the tests had from 16 to 24 chambers. The actual rate of reproduction was about fortyfold, therefore the rate of mortality to date was 34.5 in 40 or about 86 per cent. This rate of mortality is probably higher than that in the tide pools for the same period.

Turbulence and current action no doubt remove numbers of foraminifera from the beds to unfavorable areas where they cannot survive, while others become buried in the sediments to a depth from which they are unable to escape. The common shrimp *Crago vulgaris* which utilized foraminifera as its primary source of food throughout the winter, continued to eat the foraminifera, especially the smaller individuals, for several weeks after an abundance of other food had become available as may also have been the case with other bottom feeding organisms.

Empty tests removed from a sample of sediments taken in August were infiltrated with gelatine, hardened in formalin, and later decalcified, stained and cleared in much the same manner as that used in preparing living foraminifera for cytological study. When these gelatine casts were examined microscopically it was possible to detect the presence or absence of communicating passages which open between the proloculus and adjacent chambers in megalospheric tests during gametogenesis, thus making it possible to determine that portion of the population that had survived to reproduce from those which had terminated their existence from some other cause. From 100 tests so prepared 72 gave evidence of having been modified during reproductive activities and it would appear that 28 per cent, many of which were juveniles having

less than 20 chambers, had died from other causes. Cytological evidence suggests that not less than 3 per cent of the total died from parasitism by protozoa and nematodes, and that the remaining 25 per cent had either passed through the digestive tract of a metazoan organism or had died of starvation while buried in the sediments.

Evidence of a long term fluctuation in the population of the sublittoral zone is suggested by the difference in numbers of megalospheric individuals that had survived for more than one year in 1938 compared with 1939. On July 15, 1938, this amounted to 271 in a sample of 1,242 or 22 per cent, compared with 193 in 1,181 or somewhat less than 10 per cent on July 15, 1939 (Fig. 3). This difference resulted primarily from the failure of a larger proportion of megalospheric individuals to reproduce in the spring of 1939, which could have resulted from a failure in the food supply at a critical time since it would seem that reproduction is closely associated with a period of relative dormancy, followed by one of heavy feeding, and coincides with the spring outburst of phytoplankton production. In latitudes where there are only slight seasonal variations in ecological conditions and the food supply is relatively constant, reproduction may occur at any time after the foraminifera have approached a maximum growth stage.

Rate at Which the Tests are Contributed to the Sediments

The tests of *E. crispum* and its geographic subspecies are exceedingly resistant to erosion although one does occasionally find tests of which little remains other than the mass of calcium carbonate that filled the umbilical area. These eroded tests are most numerous where tidal streams cause frequent redeposition of the sediments and are seldom if ever the result of partial digestion by larger animals. Many invertebrates, including certain ecinoderms, eat enormous numbers of foraminifera, but the invertebrates which we have maintained in aquaria in order that the faecal material could be examined microscopically, had little effect upon the tests of the foraminifera which we were able to recover, and it is evident that tests of *E. crispum* or similar species pass through the digestive tract of such invertebrates as holothurians and astropectens time after time without being changed appreciably. Broken tests of both living and dead *E. crispum* were not uncommon in the surface sediments during the winter, and it would seem that these had been crushed between the mandibles of arthropods, particularly the shrimp *Crago vulgaris*, for in the digestive tract of this species, fragments of tests were frequently found.

The empty tests of the earlier developmental stages of *E. crispum* were uncommon in regions where mature individuals were most abundant, although smaller species of foraminifera inhabit these same areas. Enormous numbers of juvenile foraminifera are eaten by bottom feeding organisms and the tests of some species may be digested, but from the absence of small worn tests of *E. crispum* in the sediments it would seem that the great majority of these were removed from the beds by currents in the same manner that clay and silt were removed. These smaller tests are deposited in quiet water where they do not constitute an important part of the sediments, although we have observed deposits consisting largely of tests of minute species and the juveniles of larger species that had been transported from other areas.

It was stated that the annual rate of mortality of *E. crispum* living in tide pools was equal to the total number of individuals produced each year, and it seems probable that the number of tests washed from the tide pools that persist to become a part of the sediments of the sublittoral zone are at least equal to the maximum populations observed during the early summer. Failure to find an appreciable number of tests of the tide pool type of *E. crispum* in the sediments of the sublittoral zone suggest the extent to which they become dispersed over the floor of Plymouth Sound.

In the sublittoral zone the number of tests annually contributed to the sediments also approximates the maximum population observed including those which had survived from the preceding year. We have said that a population of 1,225 per sq. ft. was observed in Plymouth Sound and that there is little evidence that appreciable numbers of tests of this species are destroyed by having been eaten by larger organisms or through erosion except under the conditions mentioned; therefore it would seem that in the more populous areas within Plymouth Sound the number of *E. crispum* contributed to the sediments annually was well in excess of 1,000 per sq. ft. Failure to find numbers of the larger tests of this species in the muds of adjacent areas

suggests that a considerable proportion of the tests remained where they had developed. This is borne out in part by the accumulation in the area north of Drake's Island of considerable amounts of coal particles which have about the same density as the tests of the foraminifera. This coal must have accumulated over many years, because the number of coal burning ships now entering these waters is small, or more likely it may have remained in the sediments from the time when coal burners were more numerous.

In areas in which *E. crispum* are numerous an estimate of the rate of sedimentation made from the probable rate at which tests are contributed to the sediments would have little significance because turbulence and currents at these shallow depths cause much shifting of bottom materials. However the relative uniformity of populations of foraminifera over considerable areas on the floor of the sea and the slight variation in the rate of productivity that might be expected over long periods make it seem probable that the rate of sedimentation could be estimated from the rate at which tests are contributed to the sediments compared with the number of tests in core samples from the same area.

Geological Significance

The use of fossilized tests of foraminifera in making geological correlations, especially in Petroleum Geology, is well known, although it is not generally understood how small a part they have had in producing the carbonaceous material from which oil was probably derived. As we have said, a foraminifer terminates its existence by reproducing, or is eaten by some Metazoan organism, so that their only organic contribution to the sediments is in the small amount of animal matter to be found in the walls and lining of the test.

The distribution of *Elphidium crispum* and related species is limited to shallow coastal waters and the tests are common in sediments that were deposited along the coasts of Tertiary Seas. In California sandy lenses dating from the Miocene as those of the Lomitis quarry near San Pedro, or as determined from well cores from adjacent oil fields often contain large numbers of well preserved tests of this species together with fragments of *Balanus*, *Mytilus*, and other neritic material. When a foraminiferal assemblage from a similar formation includes species that tolerate water of low salinity, much fresh water must have entered the region at least during the rainy season, while the presence of species found only in warm water suggest the lower limit of temperatures in the sea, and when tests of pelagic species are numerous, typically oceanic conditions probably existed near the coast, or strong currents from such a region flowed over the beds. The tests of *E. crispum* are especially resistant to erosion so where numbers of worn tests of this species are found together with an admixture of tests of smaller and less worn tide pool or reef species, the frequent reworking of the sediments by strong tidal currents is suggested, or when mixed with species that live only in deep water it is probable that the latter had been derived from an exposure of an earlier horizon. *E. crispum* and related species do not thrive where there is an excessive amount of turbulence or current action. Where sediments consist largely of mud, the occurrence of appreciable numbers of a large species of *Elphidium* over a considerable area would suggest that they had developed in a protected shallow bay or in a sea where strong winds were of infrequent occurrence, since the only place we have taken such a species on a mud bottom was in the Java Sea where winds of gale velocity are unusual.

The foraminiferal assemblage is only one of many guides available to the stratigrapher in his attempt to reconstruct the ecology of a geological horizon, although it is perhaps the most numerous and dependable one. It is only through our knowledge of assemblages found in faunal facies along traverses extending from existing shores seaward that it is possible to interpret effectively assemblages found in geological facies. The bathymetric distribution of foraminifera in coastal waters is important to the interpretation of the paleontological record and is receiving much attention at this time. When data of this nature are published they should include oceanographic data in so far as these data are available, since this information is often difficult or impossible to obtain elsewhere (see Vaughan 1940 —and Report of the National Research Council 1941).

Summary

Several papers in the recent report of the subcommittee on the Ecology of Marine Organisms of the National Research Council May, 3, 1941 discuss the use of foraminifera in making stratigraphic correlations and in paleoecology, and

stress the need for further knowledge concerning the life activities of these marine rhizopods in the sea.

No previous attempt to deal with populations or the productivity of the foraminifera in the sea has come to our attention and although the present paper is limited in scope, the described methods offer a means of solving many problems that are of interest to students of marine sediments, both recent and geologic. *Elphidium crispum* was used to demonstrate the results of this work because it perhaps best exemplifies this group of Protozoa. Further the tests of this species when numerous in Tertiary formations are a guide to the probable depth and proximity to the coast at which the sediments were deposited and the life cycle has also provided the biologist with a classic example of dimorphism resulting from an alternation of generations.

When *E. crispum* is killed, hardened, decalcified, stained, and cleared it is possible to determine the number of chambers, the number and condition of the nuclei, whether the individual was about to reproduce, was feeding actively, or was in a dormant condition. Frequency distribution curves based on the number of chambers in both microspheric and megalospheric individuals in samples of populations taken at intervals of one month for twelve months show that in tide pools the life span is one year, and the life cycle including a sexual and an asexual phase two years, while in the sublittoral zone the life span is one or two years and the completion of the life cycle requires two, three, or even four years. Although the usual life span is the same in both life zones, the rate of growth in numbers of chambers was found to be about 40 per cent greater and the diameter of the test 60 per cent larger in *E. crispum* living in the sublittoral zone compared with those living in the littoral zone. A similar dualism was observed in other regions and in other species and may be related to the relative abundance of food in the two life zones.

The rate of reproduction of a foraminifer may be estimated from the number of progeny produced by microspheric agamonts in cultures and the ratio of megalospheric to microspheric individuals in the sea. In *E. crispum* the annual rate of productivity for the two generations was found to be about thirtyfold and fortyfold respectively in the littoral and sublittoral zones. The population per unit area on the floor of the sea was found to exceed 1,000 per sq. ft. in some regions and was obtained from the number of protoplasmic casts in samples of sediments taken with a special suction grab after the sample had been treated with formalin and the tests dissolved in acidified alcohol. It was found that the tests of *E. crispum* are not destroyed in passing through the digestive tract of the invertebrates studied, and usually show slight evidence of erosion except where the sediments are shifted about by tidal streams and storm waves. Since reproduction terminates the life of the individual and both sexual and asexual reproduction is for the most part limited to a brief period in the spring, the annual productivity of the species can be easily determined.

If the number of tests contributed to the sediments annually were determined for several species living on a more stable bottom and these numbers were compared with the numbers of tests in core samples from the same area it would be possible to estimate the rate of sedimentation.

Foraminifera usually terminate their existence by reproducing or are eaten by larger organisms, and the empty tests of most species contain little organic matter. Although the use of the tests in petroleum geology is well known, it is not so generally understood how small a part they have had in the production of petroleum, however, the population of benthonic foraminifera in the sea is related to the amount of food in the water and in turn to the amount of organic matter in the sediments.

Acknowledgments

This is the second of a series of papers made possible through a John Simon Guggenheim Memorial Foundation Fellowship, 1938–39, and 1939–40, and a Rhumphius Fellowship granted by the Treub Laboratories of the Netherlands East Indies, 1940–41. Much of the technical work was done by my wife, Ethel, who also assisted with the manuscript. All samples were collected personally or under personal supervision, and have been preserved for further reference.

We wish to express our appreciation to those of the staff of the Marine Biological Association at Plymouth and the Oceanographic Laboratory at Woods Hole who have helped to make the work possible, and especially we wish to acknowledge the assistance of Captain "Bill" Searles of the *Gammarus* who also helped J. J. Lister collect material at Plymouth for his monograph on the Foraminifera, 1895.

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PLATE I

Figs. 1, 4, 8 and 11, ×76; Figs. 2, 3, 5, 6, 7, 9 and 10, ×220

FIG. 1. Horizontal section through a fifty seven chambered megalospheric gamont showing the arrangement of the chambers and the position of the nucleus.

FIG. 2. Megalospheric nucleus showing many small endosomes and a large one that is vacuolated.

FIG. 3. Horizontal section through a thirteen chambered megalospheric juvenile in which the nucleus has remained in the proloculus.

FIG. 4. Cross section through a megalospheric gamont showing retral processes, foramina, and canals.

FIG. 5. Horizontal section through early chambers of a megalospheric gamont showing that only one other chamber communicates by a foramen with the proloculus.

FIG. 6. Similar to the above except that the several chambers surrounding the proloculus now communicate by secondary foramina with this primordial chamber. The presence of many minute nuclei in all chambers attests that gametogenesis is well advanced.

FIG. 7. Similar to Fig. 6, except that multiple fission followed by binary fission of the resulting gemetocytes has produced mononucleate gametes which occupy all chambers and the interseptal and spiral canals.

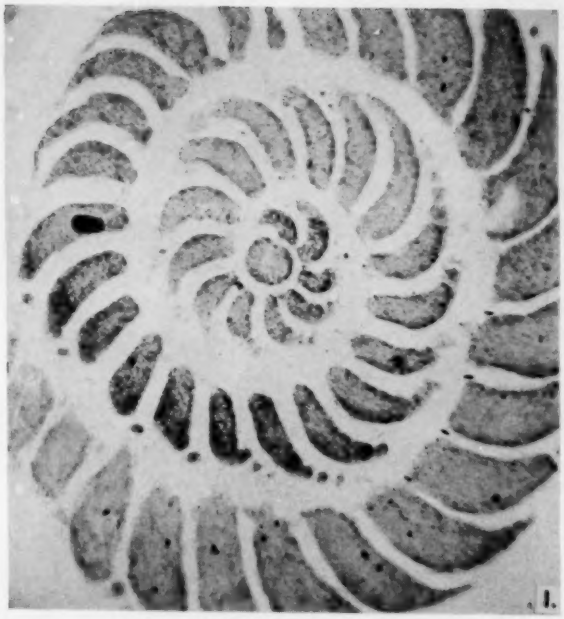
FIG. 8. Microspheric individuals showing shrinkage of the cytoplasm typical of winter dormancy. This condition was not observed at any season in warmer climates.

FIG. 9. Section showing four nuclei typical of vegetative microspheric agamonts.

FIG. 10. Early chambers of a microspheric agamont.

FIG. 11. Horizontal section of a microspheric agamont showing the arrangement of the chambers and the distribution of the many nuclei.

PLATE I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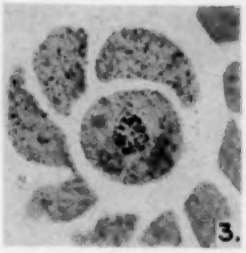
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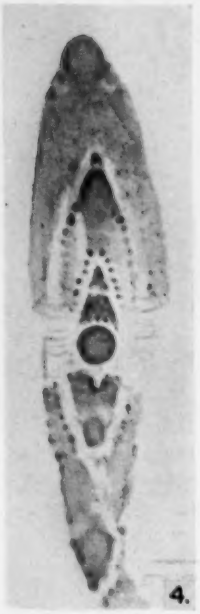
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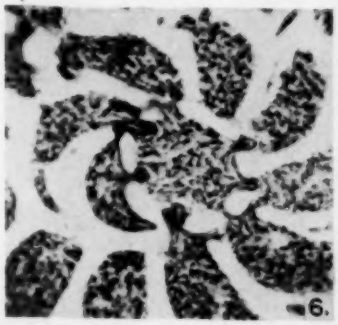
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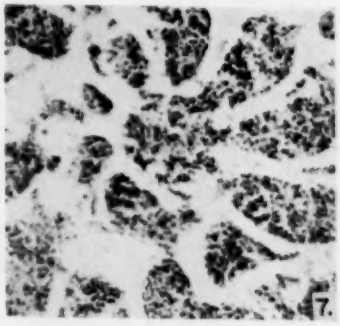
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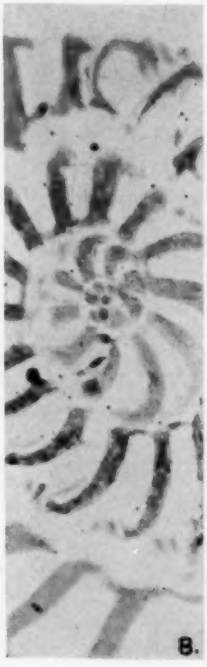
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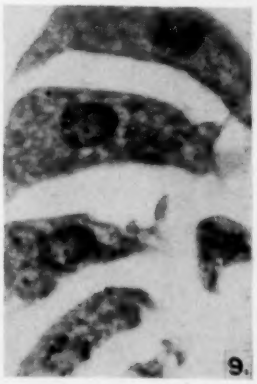
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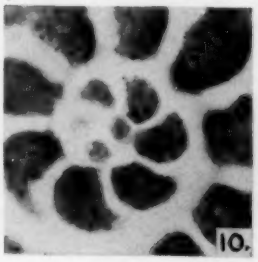
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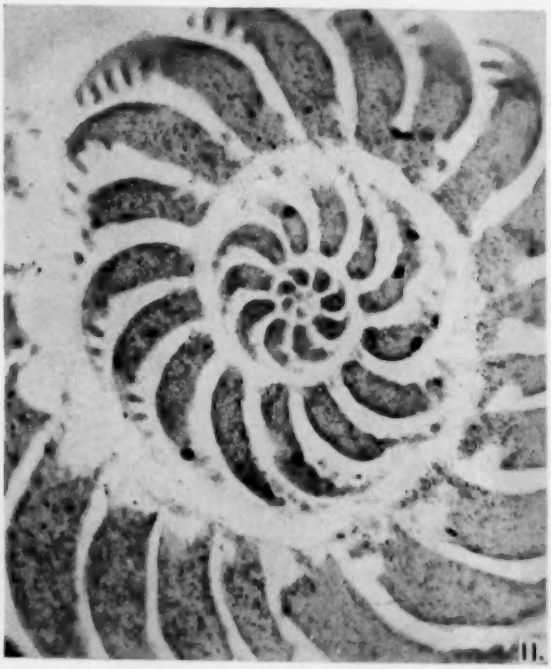
11.

THE HEALTH OF REGISTRANTS AND THE PRESIDENT'S PLAN OF REHABILITATION

LEONARD G. ROWNTREE, Med.-Res.

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(Read November 22, 1941)

SELECTIVE Service is charged with the function of procurement of men for the Army. More than 17,000,000 men have been registered to date and more than 2,000,000 have been examined and 1,000,000 men inducted into the Army.

Selective Service reveals the average registrant as 25.7 years of age, 5 feet 7½ inches tall and weighing 150 pounds. This compares favorably with the average draftee of World War I, who was also 5 feet 7½ inches tall, but who weighed only 142 pounds. The average for the soldier of the Civil War was 136 pounds, or 14 pounds less than that of the average registrants today. His average height was 5 feet 7½ inches. It appears, therefore, that the height of our young men is unchanged during the last eighty years, but that the weight is somewhat increased. This latter fact is not in keeping with the belief held by many that malnutrition is exceedingly prevalent among American citizens. It is difficult, under existing conditions, to reconcile this increase in weight with widespread want of food or serious lack of nutrition.

The average height of registrants qualified for general military service was 68.1 inches and the average weight was 152 pounds. The height of registrants examined varied from 54 inches to 88 inches, and 98.3 per cent were between 60 inches and 78 inches, which are the minimal and maximal heights, respectively, for acceptance by the Army.

Registrants varied in weight from 85 pounds to 385 pounds and 93 per cent were between 100 pounds and 190 pounds. The minimum weight for admittance into the Army for registrants 60 inches tall is 105 pounds and the standard weight for a registrant 78 inches tall is 184 pounds.

Selective Service examinations also reveal that great differences exist in the health as recorded in various parts of the country. A "Health Belt" is found in a group of western States centering around Colorado. A somewhat similar belt of good health was found during the last war. The poorest physical condition is encountered in some of the southern States. The difference in health in some of these areas is quite marked; thus, for every 10 men examined in Colorado, 7 are found fit for service, whereas in one of the southern States, only 3 are acceptable. It must be realized, however, that these figures relate solely to the results of the examination, and do not cover the question of resistance or immunity to disease. In the last war, during epidemics of disease, it was found that men from these healthy areas were as susceptible, or even more susceptible at times, than the young men from other parts of the country.

THE RESULTS OF PHYSICAL EXAMINATION

Of the 2,000,000 young men throughout the length and breadth of the land who have undergone careful physical examination at the hands of Selective Service and the Army Induction Boards, one million have fallen short of Army requirements. One hundred thousand (100,000) have been rejected for varying degrees of illiteracy and 900,000 for physical and mental deficiencies, defects, disorders or diseases. Four hundred and thirty thousand (430,000) have been classified IV-F as unfitted for any form of military service, and 470,000 as fit only for limited service. Thus 50 per cent of the registrants to date have fallen short of the standard requirements for service with the military forces.

These figures are not to be interpreted as representing 50 per cent invalidism of the registrants examined, or as indicative of 50 per cent illness of the population in general. They simply represent unfitness for military service according to the standard requirements that have been set up by the Army.

Apparently, Selective Service has uncovered a vast field of physical unfitness. The registrants concerned, in the majority of instances, are not ill; they are not sick; they are at work and carrying on satisfactorily despite their defects and deficiencies. They suffer from hidden potential sources of disease and cover up as they carry on. Defects from which they suffer today, minor as they may be, may prove responsible for serious

illness in the years to come. If these defects are serious enough to render the individual unfit for military service, they undoubtedly are playing at least some role in the efficiency of the individual's carrying on his ordinary occupations in civilian life.

To some, the state of physical unfitness appears a matter of minor importance. To the Army, it is a matter of major significance since it is cutting down manpower for military service to the extent of at least 50 per cent. To the nation, this unfitness comes as a decided shock. It constitutes a national weakness heretofore not properly appreciated and calls for immediate correction from the standpoint of securing manpower for the Army, and constitutes a challenge to the nation for the program of building up of physical fitness.

When we come to a breakdown of the various causes for rejection, we find them listed as follows (Table 1):

TABLE 1

THE PHYSICAL REJECTIONS OF REGISTRANTS

Cause	Number of Cases	Percentage
Dental defects	188,000	20.9
Defective eyes	123,000	13.7
Cardiovascular diseases	96,000	10.6
Musculo-skeletal defects	61,000	6.8
Venereal diseases	57,000	6.3
Mental and nervous diseases	57,000	6.3
Hernia	56,000	6.2
Defects of ears	41,000	4.6
Defects of feet	36,000	4.0
Defective lungs, including tubercolosis	26,000	2.9
Miscellaneous	159,000	17.7
	900,000	100

INCIDENCE OF DEFECTS

The major pathology indicates the reasons why registrants were rejected, but does not afford an accurate index as to the incidence and prevalence of diseases and defects among them. In this study a maximum of three defects was recorded. A total of 27,031 defects were tabulated from the 19,923 reports of physical examination, an average of 1.4 defects per registrant examined. No defects were recorded, however, for 5,741 registrants, or 29 per cent of the total number examined. Of the total of 27,031 defects one or more was recorded for each of 14,182 registrants, an average of 1.9 defects per registrant with defects. Two defects were recorded for each of 8,433 registrants and three defects for 4,416 registrants.

In Table 2 there is a list of the defects or diseases tabulated by broad classifications with the rate per thousand registrants examined.

TABLE 2

INCIDENCE OF DEFECTS FOUND IN 19,923 REGISTRANTS EXAMINED BY SELECTIVE SERVICE LOCAL BOARDS

Defect or Disease	Defects Found	Defects Per 1,000
Eyes	2,305	115.7
Ears	887	44.5
Teeth	2,795	140.3
Mouth and gums	1,273	63.9
Nose	1,372	68.9
Throat	1,321	66.3
Lungs	327	16.4
Tuberculosis	114	5.7
Cardiovascular system	2,000	100.4
Blood and blood-forming organs	19	1.0
Hernia	1,287	64.6
Kidneys and urinary system	279	14.0
Abdominal viscera	244	12.2
Genitalia	1,175	59.0
Venereal	695	34.0
Skin	2,308	115.8
Hemorrhoids and rectal defects	610	30.6
Varicose veins	531	26.7
Mental and educational deficiency and illiteracy	239	12.0
Mental disorders	362	18.2
Neurological	454	22.8
Musculo-skeletal	2,018	101.3
Feet	2,888	145.0
Endocrine disturbances	319	16.0
Tumors	228	11.3
Infectious, parasitic and epidemic diseases	7	0.4
Other diseases and defects	974	48.9
Total	27,031	1,356.8

This tabulation includes defects which do not disqualify as well as defects which do disqualify for general military service. Defective feet accounted for the largest number of diseases and defects recorded for any single organ, section or system of the body and comprised 10.7 per cent of the total number of defects tabulated. Dental defects, which was the largest cause of rejection for military service, comprised 10.3 per cent of the diseases and defects. In addition to non-disqualifying defects, a large proportion of the disqualifying defects are minor insofar as health conditions are concerned. Many defects are a cause for rejection for service in the Army, but in no way hinder the performance of many civilian occupations.

Effect of Age on Selection and Rejection

Inasmuch as the reports of physical examination considered in this survey were for men examined prior to May 31, 1941, registrants between the ages of 21 and 36 were included as well as a small number of men between the ages of 18 and 21 who volunteered through the Selective Service System for military service. Two-thirds of the registrants examined by local boards were between the ages of 21 and 28. Registrants between the ages of 28 and 36, inclusive, accounted for 31.3 per cent of the total number examined, and the volunteers between the ages of 18 and 21 accounted for 2.1 per cent of the total registrants examined.

The rate of rejection of registrants between the ages of 31 and 36 was nearly twice as great as that between the ages of 21 and 25, inclusive. Sixty-one per cent of the registrants between the ages of 31 and 36 were unacceptable for general military service as compared to 45 per cent between the ages of 26 and 30, and 34 per cent between the ages of 21 and 25. The percentage who were qualified for general military service varied from 70.5 per cent for registrants 21 years old to 29.9 per cent for registrants 36 years old at the time of physical examination. The relationship that exists between the registrants' age and availability for general military service is shown in Table 3 and Chart II.

TABLE 3

Age of Registrants by Availability for Military Service

Age[1]	Number Examined	Percentage of Total Examined	Qualified for General Military Service		Qualified for Limited Military Service		Disqualified for Military Service	
	Number	Per Cent	Number	Per Cent	Number	Per Cent	Number	Per Cent
18	153	0.8	129	84.3	8	5.2	16	10.5
19	143	0.7	118	82.5	13	9.1	12	8.4
20	127	0.6	94	74.0	17	13.4	16	12.6
21	2,083	10.4	1,469	70.5	347	16.7	267	12.8
22	2,688	13.5	1,810	67.3	511	10.0	367	13.7
23	2,248	11.3	1,504	66.9	393	17.5	351	15.6
24	1,372	9.4	1,222	65.3	343	18.3	307	16.4
25	1,692	8.5	1,034	61.1	331	19.6	327	19.3
26	1,492	7.5	905	60.7	276	18.5	311	20.8
27	1,189	6.0	682	57.4	268	22.5	239	20.1
28	1,071	5.4	576	53.8	260	24.3	235	21.9
29	839	4.2	435	51.8	213	25.4	191	22.8
30	869	4.4	400	46.0	226	26.0	243	28.0
31	697	3.5	306	43.9	188	27.0	203	29.1
32	681	3.4	282	41.4	188	27.6	211	31.0
33	669	3.4	282	42.4	188	28.1	199	29.7
34	602	3.0	212	35.2	159	26.4	231	38.4
35	527	2.6	176	33.4	172	32.6	179	34.0
36	157	0.8	47	29.9	54	34.4	56	35.7
Unknown	124	0.6	71	57.2	29	23.4	24	19.4
Total	19,923	100.0	11,754	59.0	4,184	21.0	3,985	20.0

[1] Age at last birthday at time of physical examination

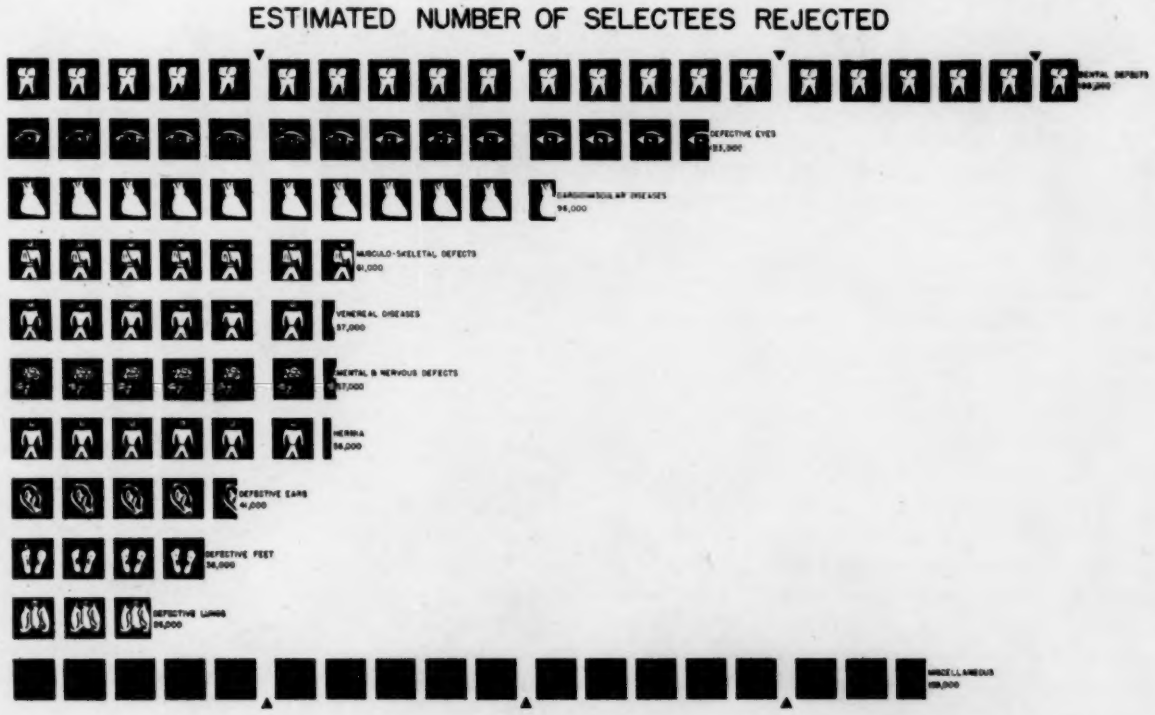

Chart I. Each symbol represents one per cent of the 900,000 rejected.

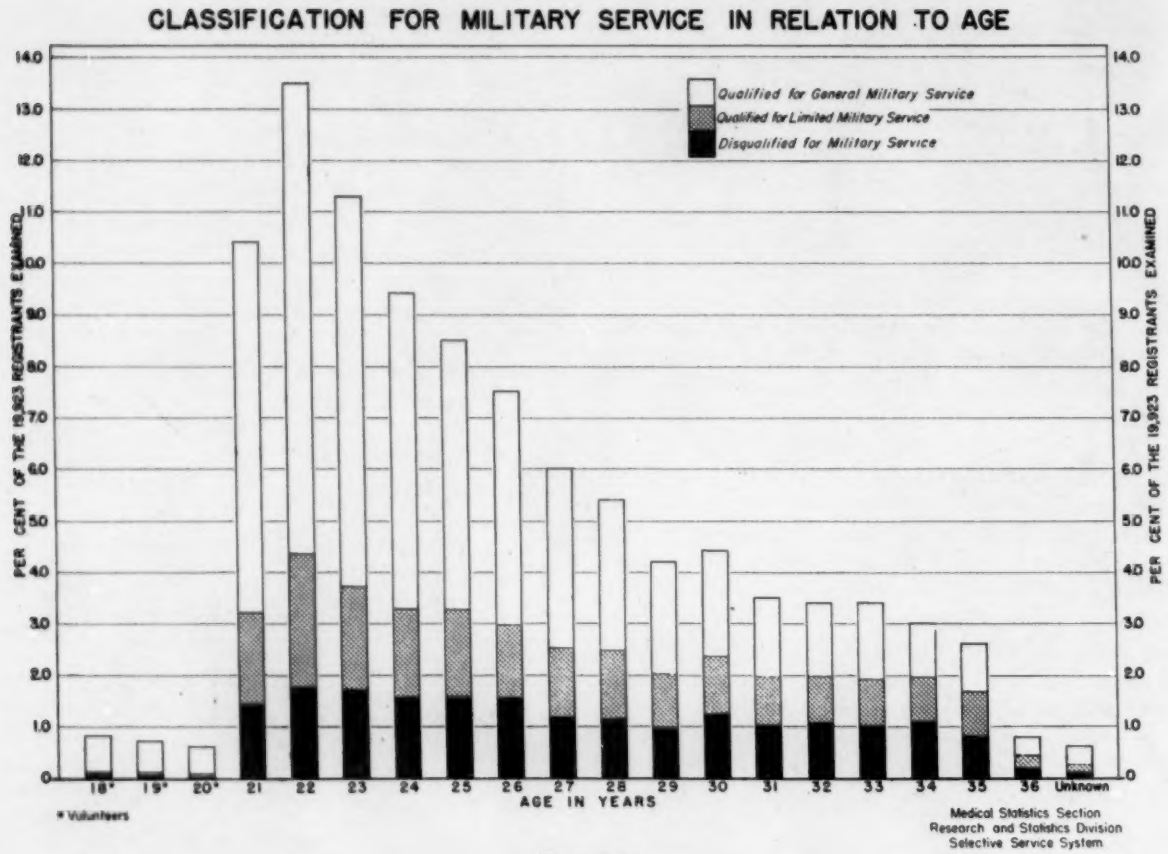
CLASSIFICATION FOR MILITARY SERVICE IN RELATION TO AGE
Qualified for General Military Service
Qualified for Limited Military Service
Disqualified for Military Service
PER CENT OF THE 19,923 REGISTRANTS EXAMINED
AGE IN YEARS
18* 19* 20* 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 Unknown
* Volunteers
Medical Statistics Section
Research and Statistics Division
Selective Service System

CHART II

OCCUPATION

About one-seventh of all registrants examined were unemployed at the time of examination. The largest group of employed registrants physically examined were operatives and kindred workers comprising about 20 per cent. Most numerous among operatives and kindred workers were chauffeurs, truck drivers, delivery-men, weavers, knitters, spinners, assemblers, and operators of lathes, drill presses and other machines. Farmers and farm laborers, the second largest employed group, accounted for 14 per cent of the total number examined. Craftsmen and foremen, which accounted for 8 per cent of all registrants examined, were mostly carpenters, cabinet makers, pattern makers, mechanics, repairmen, machinists, millwrights, tool makers, painters, paper hangers and printers. Approximately one per cent of the registrants examined were students. The occupations of the registrants examined are shown by broad classifications in Table 4.

TABLE 4

OCCUPATIONS OF 19,923 REGISTRANTS PHYSICALLY EXAMINED

Occupation	Number of Registrants Physically Examined	Percentage of Total Examined.
Professional workers	590	3.0
Semiprofessional workers	226	1.1
Farmers (owners, tenants and croppers) and farm managers	1,099	5.5
Proprietors, managers, and officials except farm	779	3.9
Clerical and kindred workers	1,548	7.8
Salesmen	878	4.4
Craftsmen, foremen and kindred workers	1,679	8.4
Operatives and kindred workers	4,064	20.4
Domestic service workers	56	0.3
Protective service workers	64	0.3
Service workers, except domestic and protective	936	4.7
Farm laborers and foremen	1,683	8.4
Laborers, except farm and mine	2,487	12.5
Nonclassifiable returns	571	2.9
Emergency workers and unemployed	3,042	15.3
Students	221	1.1
Total	19,923	100.0

Race, Urban-Rural Residence, and Place of Birth

The ratio of Negro registrants to the total number examined was approximately the same as the ratio of Negroes to the total population, 11 per cent. Registrants of other races, which accounted for less than one per cent of the total number examined, were mostly Chinese, Japanese and Indians. The rate of rejection for Negro registrants was 42.6 per cent as compared to 40.8 per cent for white registrants. Two-thirds of the registrants examined were from urban communities. The rate of rejection for registrants from urban areas was 42.4 per cent as compared to 38.1 per cent for rural areas.

The United States and its territories was the place of birth of 97.3 per cent of the registrants examined, most of the others having been born in Canada, British Isles and Europe. The rate of rejections for native born registrants was 40.9 per cent as compared to 45.6 per cent for registrants born outside the United States and its territories. This, perhaps, can be accounted for by the differences in age groups.

Rehabilitation of Registrants

Because of the relatively poor physical state of 50 per cent of the registrants examined and because of the urgent need of manpower for the military forces, the President called a conference at the White House on October 9, of this year to determine what measures should be instituted for the correction of this situation.

The President's Plan for Rehabilitation provided for the correction of remediable defects of 200,000 registrants, the program to be carried out by Selective Service through its existing boards and personnel, and to be effected in the registrant's home town under the supervision of Selective Service in conjunction with the family doctor and dentist, and County Medical and Dental Societies, the professional work to be done at a reasonable cost for which funds are to be made available.

The rehabilitation of 200,000 men represents a large order. Success demands the best services and the closest coöperation of the doctors and dentists and the clinics and hospitals throughout the entire country, the project calling for the maximum efficiency of the two professions concerned. Rehabilitation should be effected with the minimum risk to the registrants. Hence, Selective Service has adopted "safety first" as its rehabilitation slogan. Such a program calls for careful planning, wise selection of candidates and skillful handling of them in the application of remediable measures.

For Army purposes, all defects cannot be regarded alike relative to satisfactory rehabilitation. The Army, through long years of experience, has determined what diseases and disorders readily lend themselves to successful rehabilitation that produces "fit men" who can stand up satisfactorily under the rigorous demands of Army training and service. Hence, the Surgeon General of the Army, James C. Magee, has been consulted freely as to preference in the types of defects to be remedied. But for conservation of effort and expense, still more is required. It is necessary to know in advance, before rehabilitation is actually attempted, which individual registrant will be acceptable to the Army when the remedial measures have been applied. Hence, arrangements have been perfected whereby Army examination given prior to rehabilitation will determine and certify, so to speak, subsequent acceptance of the individual after his rehabilitation has been completed. This Plan aims to facilitate the President's objective quickly to rehabilitate a large number of suitable registrants who will be immediately inducted into the military forces, thereby actually increasing the manpower of the country.

Since dental defects are responsible for the greatest number of rejections in America, and since they lend themselves readily to remedy and because the risk of remedy is exceedingly remote, it is planned to rehabilitate 100,000 victims of dental disorders.

Among the medical deficiencies and defects, the following groups will lend themselves readily to rehabilitation: deficiency in vision, which has not been adequately corrected; certain cases suffering from conjunctivitis, pterygium, or from scars of deformities of the lids; among cardiovascular diseases, many may be salvaged who are found to present borderline hypertension, mild tachycardia, functional murmurs, extra systoles, low blood pressure; among musculo-skeletal defects, many should be reclassified, especially those who complained of aches and pains of various kinds at time of examination, who, in the interim have not found it necessary to be under the care of a qualified physician; likewise, many cases of first degree flat feet can be salvaged.

A large number of individuals who were found suffering from venereal diseases, particularly un-

complicated gonorrhea, may, by this time, have recovered, and reclassification may find them acceptable for induction.

Among the miscellaneous group, there are undoubtedly a very considerable number of disorders contributing little to morbidity or mortality, yet easily susceptible to correction. In this category are many suffering from underweight, overweight, from tonsilitis, nasopharyngitis, nasal polypi, deviation of the septum. Many others suffering from moderate degrees of skin diseases, acne or furunculosis and others with external hemorrhoids or presenting evidence of postural or orthostatic albuminuria, transient glycosuria, phimoses, etc.

Another group of diseases is quite unfavorable from the standpoint of rehabilitation. This group includes the victims of nervous and mental disorders, uncorrectable defects of vision or hearing and patients who have suffered chronic diseases of the lungs, especially tuberculosis and victims of syphilis. From these groups, it will be impossible to find many who can be rehabilitated to meet the requirements of the military forces.

For the present at least, a conservative position is being taken towards patients suffering from conditions requiring surgical interference, such as hernia, varicocele, etc. Army experience has established the fact that more is needed in many of these cases than the mechanical cure. Surgical risk attends all forms of major operation, and to a slight extent, even minor surgical measures. Surgery should not be employed in this program except on request of the registrant and, for reasons which are obvious, with the assumption by him of full responsibility for the surgical risk. The thought must also be given to possible development of untoward psychical reactions which may prove detrimental to satisfactory Army service. Selective Service is therefore giving very careful consideration to the different types of rehabilitation in order that the greatest benefit may accrue to all persons interested, the registrant, the Army and the national health in general.

Prehabilitation

Rehabilitation takes cognizance of defects at the time of examination by Selective Service or at the Army examining station. There is, however, another procedure whereby these defects can be discovered and corrected in advance of the official examination. For this plan, Selective Service has suggested the term Prehabilitation.

Prehabilitation provides that: (1) through education of the registrants, the standard requirements of the Army are made known; (2) that registrants falling short of these requirements consult their family doctor and dentist; (3) have their remediable defects corrected; (4) be given a certificate to this effect to present to their examining boards. The plan of Prehabilitation has already been published covering some five articles that have appeared in the *Journal of the American Medical Association*, *Journal of the American Dental Association* and other Journals.

In many respects, Prehabilitation is a form of preventive medicine, especially the correction of minor defects before they develop into serious etiological factors such as might be concerned in the pathogenesis of more serious types of disease. Prehabilitation has been employed in the University of Minnesota to render male students of military age fit for examination and for service.

Conclusion

The facts revealed by Selective Service relative to the widespread prevalence of defects, deficiencies, disorders and diseases among registrants should exert a salutary effect on the Nation as a whole. Once their nature, prevalence, distribution and their menace to health and efficiency are disclosed, they are more apt to receive appropriate attention and thereby be eradicated. There is no doubt that the correction of these defects as they develop would contribute largely to the elevation of public health onto a higher plane. ***Better Public Health can be attained only through Better Individual Health. Better Individual Health can be attained only by the prevention of disease and the correction of minor defects as they arise.***

THE MYSTERY OF COR CAROLI

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ABSTRACT

Cor Caroli, or α^2 Canum Venaticorum, has a spectrum of type A, with fairly strong lines of *H*, *Mg* II, *Fe* II, *Cr* II and with a temperature of about 10,000°. Its spectral absorption lines show unusual changes in intensity, and until recently the star was believed to be unique in this respect. Several other A-type stars are now known which show similar changes. The spectral lines of α^2 Can Ven are arranged in three groups (the designations of which must not be confused with the designations of spectral type or temperature). *Group A* consists of *Eu* II, *Gd* II, *Dy* II and other rare earths, *Fe* I, *Ti* II, etc. The lines of *Eu* II show a variation in equivalent width by a factor of at least 10; the other elements, especially *Ti* II, *Fe* I, *Ce* II, *Sa* II, show smaller ranges. The period is constant, P = 5.46939 days. *Group B* consists of several elements, such as *H*, *Si* II, *Fe* II, and *Cr* II, which vary in the opposite sense of group A lines. The range is greatest for *Cr* II. *Group C* consists of lines which remain constant in intensity. Examples are *Mg* II and *Sr* II.

The lines of groups B and C are essentially constant in velocity. The strong lines of *Eu* II show large negative displacements from about 1.6 days prior to their intensity maximum, and large positive displacement until about 1.5 days after intensity maximum. During the remaining 2.4 days the lines are so weak that blends may interfere with the measurements. The lines of all groups probably originate in the star, but not necessarily in the same atmospheric layer.

The great strength of the rare earths, especially *Eu* II, at certain phases is very strange. The variation in intensity cannot be explained in terms of Saha's theory of thermal ionization, because *Sr* II, the ionization potential of which is only 0.2 volt lower than that of *Eu* II, shows no appreciable change in intensity.

The phenomena strongly suggest that departures from thermodynamic equilibrium are operating in such a way as to produce a variable amount of suppression of double ionization near 11.2 volts, without appreciable change at 11.0 volts (*Sr* II) and with a permanent enhancement of ionization at 11.8 volts (*Ca* II). The nature of this phenomenon is not clear, but it is correlated with the variable velocities of the *Eu* II lines, and may involve a conversion of the energy of ionization into kinetic energy of motion. It is significant that the continuous spectrum of an A-type star is greatly distorted from that of a black body by the Lyman lines and the Lyman continuum. It is from the region between L_α and L_β that the ionization energy of the rare earths is derived, while the region of L_β contributes especially to the ionization of *Ca* II. A strong emission line at L_β would perhaps explain the excessive ionization of *Ca* II.

IT was reported by Sir Charles Scarborough, court physician of King Charles II, that the star now known as α Canum Venaticorum had shone with unusual brilliance on the night of the King's return to London on May 29, 1660. In order to commemorate the event, which appeared to give divine sanction to the restoration of the Stuart dynasty in England, the Astronomer Royal, Edmond Halley, in 1725 designated this star as the center of a small constellation, the Heart of Charles, and since that time the star itself has often been called Cor Caroli.[1] The story of the brightening of α Can Ven has never been verified. It was not regarded as a suspected variable in the catalogues of Gore and of Zinner, although more recently it has been found to have a periodic variation of very small amplitude.

Our present interest in Cor Caroli depends upon the fact that it is the typical representative of a mysterious class of stars having variable absorption lines in their spectra. We know very little about these stars. Half a dozen, or so, have been discovered, mostly by W. W. Morgan at the Yerkes Observatory. But the periods are known for only two or three, and the character of the variation, or its causes, remain to this day an unsolved puzzle.

α Can Ven is a visual double star. The distance of almost 20″ between the 2.9 mag. primary (designated as α^2) and the 5.7 mag. secondary (designated as α^1) has remained constant since 1830. So has also its position angle of 228°. Both components are usually listed as spectroscopic binaries. The fainter shows a rather normal spectrum of type F. Harper found a range in the radial velocity of 17 km/sec and I have obtained similar results from spectrograms taken at the Yerkes Observatory. So far as I know no orbit of this star has been published. Incidentally, the wide separation makes it possible to guide with the slit-spectrograph on each component without any danger of contaminating the plate with the light of the other.

The bright component was originally announced as a spectroscopic binary by Belopolsky. But in the last catalogue of spectroscopic binaries from the Lick Observatory, J. H. Moore lists it

[1] R. H. Allen, *Star Names and their Meanings*, New York, 1899. I am grateful to Mr. Herbert W. Cornell of Milwaukee for this reference.

among "stars having variable radial velocity, but which are not binaries." [2]

The proper motion of the system is 0".233 in position angle 282°. The parallax is 0".024 and the mean radial velocity is − 3.5 km/sec.

The spectrum was first definitely classed as peculiar in 1897 by Miss Maury.[3] She called attention to the weakness of *Ca* K and the great strength of *Si* II 4128, 4131 in this star of type A (according to our present Draper classification). She adds: "Some of the fainter lines appear to be peculiar in wave length, while others differ in intensity from the corresponding lines of the stars in division *a* (of her group VIII). In the weakness of the line K alluded to, this star resembles those of composite type, and its peculiarities may perhaps be due to its known duplicity. If, however, in the companion star the line K is so faint, it is difficult to explain the absence of the Orion lines."

We now know that neither the distant companion (which does not affect the spectrum in a slit instrument), nor any hypothetical spectroscopic companion of the star α^2 can produce the observed effect. The phenomenon is one frequently observed among A-type stars, as has been shown by Morgan.[4]

The peculiarities noted by Miss Maury were confirmed in 1906 by Lockyer and Baxandall. They also found that "the more pronounced lines of iron, titanium, and chromium are present, but weak," while *Mg* 4481 is strong. "The ordinary metallic arc lines, which occur prominently in the lower type stars, are lacking. Some of the strange lines, that is, lines not in the normal spectrum of the same general type, appear to be identical with those in θ Aurigae. Thus there are lines of which the wave lengths are approximately 3954.3, 4076.5, 4136.3, 4192.0, and 4376.8. . . . There are no lines of considerable intensity peculiar to α Can Ven, though some of the fainter lines seem to be peculiar to this spectrum." [5]

Incidentally, even now, we know little about the lines of Lockyer and Baxandall:

L.-B.	Int.	Modern Identification
3954.3	2	Unknown
4076.5	6	Mostly unknown, *Si* II 6.78(1); *Fe* II 7.16(3)
4136.3	2	Unknown
4192.0	3+2	Blend of *Si* II and *Ce* II
4376.8	3+1	Blend of *Sm* II and *Ce* II

[2] *Lick Obs. Bull.* No. 483, 1936.
[3] *Harvard Annals*, **28**, 96, Remark 69, 1897.
[4] *Pub. Yerkes Obs.*, **7**, Pt. 3, 1935, see especially Plate I.
[5] *Proc. Royal Soc.* London, **A77**, 550, 1906.

This shows how little progress has been made in this field in the past 36 years!

The star became of particular interest to astronomers in the latter part of 1906, when Ludendorff [6] announced that the lines of α^2 Can Ven vary in intensity. He wrote "A comparison of the best plates proved with a fair degree of certainty that the intensities of a number of lines are really variable." He mentions especially the following lines:

Ludendorff	Modern Evidence
Cr 4341.93 on some plates strong, on others not visible	*Fe* II 4351.76 of group B or C, but always easily visible
Fe 4472.88 sometimes strong, at other times absent	*Fe* II 4472.92, group C
Fe 4238.97 sometimes strong, at other times absent	*Fe* I 4238.82, group C, always visible
Mg 4481 sometimes sharp, at other times less so	*Mg* II 4481, group B, but always very strong

In view of the remarkable results obtained by Belopolsky and all later observers it is surprising, to say the least, that Ludendorff picked out precisely those lines which vary little. He either did not notice those lines which show enormous changes in intensity, or they were not in the spectrum at the time of his observations. It should be remembered, however, that the most conspicuous lines of variable intensity, namely λλ 4129, 4205 were probably beyond the range of his instrument. Ludendorff found no regularity in the variation. It must be admitted that the discovery of a star with variable lines was a remarkable result, especially in 1906, when hardly any other star of similar character was known. Ludendorff's work represents the first step in a long series of papers which were devoted to α^2 Can Ven.

The first, and perhaps most significant, of these investigations is one by A. A. Belopolsky at Poulkovo.[7] He discovered the important group of variable lines now attributed to "group A," among which are λλ 3984, 4038, 4130, 4291, 4386, etc. From the intensities of λ 4130 he derived the period

$$P = 5.50 \text{ days.}$$

He also found a second group of lines, now designated as "group B," which vary in the opposite sense as do the lines of group A and, finally, established that several lines do not vary at all

[6] *A.N.*, **173**, 4, 1906.
[7] *A.N.*, **196**, 1, 1913.

or vary but little. Among the latter ("group C") he included *H*, *Mg*, *Ca* and *Fe*. We must be careful not to confuse the designations of the groups A and B with the names of the spectral types of the stars. α Can Ven is an A-type star and it has lines of three groups: A, B and C.

From the group A line λ 4205 Belopolsky derived a velocity curve with a considerable range, but the lines of group C and many of group B showed no conspicuous changes in velocity. Line λ 4130 agreed with λ 4205 when the former was visible, but at times of minimum intensity it disappeared completely; λ 4205 was always visible, but occasionally became very weak and sometimes double; when that happened only one component was believed to belong to the star proper.

Belopolsky concluded: "It is difficult to decide wherein to seek the cause of this phenomenon. An obvious hypothesis suggests itself, namely, that the central body is surrounded by a gaseous satellite or a gaseous ring having a condensation of matter at one point. This hypothesis is supported by the sign of the variable velocities (negative velocities preceding maximum of intensity of λ 4130 and positive velocities following maximum), but the details of the observations still present difficulties which may perhaps be cleared up after more material has been accumulated."

Belopolsky's work [8] was very diligently continued by his pupils at Poulkovo, especially by Rossovskaya, Gerasimovič and Markov.[9] These investigations greatly added to our knowledge of the variable lines, but gave no new concepts and no further tests of the hypothesis. Belopolsky himself attempted in 1915 to measure the line λ 4205 as a double and to derive an orbit for two visible components. But λ 4130 does not show the doubling and Coudé plates of high dispersion now rule out the double velocities. Yet, one aspect of Belopolsky's curves is of importance. He places the minimum velocity of the stronger component about 1.2 days earlier than the maximum of intensity of λ 4130. The curve is thus very unsymmetrical.

Almost immediately after the first paper by Belopolsky had appeared, Baxandall [10] pointed out that five of Belopolsky's variable lines of group A are due to europium. The surprising fact that this rare-earth element should be conspicuous in an A-type spectrum was commented upon by various speakers at the meeting of the Royal Astronomical Society at which Baxandall's paper was presented. A. Fowler suggested that [11] "slight disturbances in the star might bring about small changes of level sufficient to put the europium in or out of the absorbing region, and so determine the presence or absence of the lines."

Belopolsky's brilliant discovery aroused a great amount of admiration, as well as of speculation. It was thought by Stratton [12] not only "to round off Belopolsky's discovery of the binary nature of the Cepheid variables by finding one double whose components have different and visible spectra" (namely, groups A and B of the variable lines), but that the lines which neither shift nor vary present "the problem of Hartmann's fixed calcium lines all over again, complicated by the presence of several elements." Hydrogen, silicon, iron and magnesium were thought by Stratton to belong to "an envelope surrounding and moving with the system . . . (of) both components of the binary."

The next significant spectroscopic study of α Can Ven was that by C. C. Kiess at the Observatory of the University of Michigan.[13] The star was placed upon the observing program immediately after Belopolsky's announcement in 1913 and a large number of plates were measured by Kiess for radial velocity and for the identification of lines. His table of wave lengths gave the first complete study of this interesting spectrum and served as the starting point of most later studies. The velocity curve shows the same general trend as the one of Belopolsky, but only one component was measured. Kiess stated that "maximum positive displacement occurs when the line is of minimum intensity; and maximum negative displacement when it is of maximum intensity." This is contrary to Belopolsky, who found a lag of 1.2 days between the two epochs. However, Kiess' paper does not contain a study of the line intensities, and the question arises whether he determined the epoch of maximum intensity with any precision. Fortunately for us, Miss G. Farnsworth was permitted some years ago to examine the Michigan spectrograms. With the help of this material and the published data from all other observatories she found that all epochs could be repre-

[8] *Bull. de Poulkovo*, No. 70, 1915.
[9] *Ap.J.*, **72**, 301, 1930.
[10] *Observatory*, **36**, 440, 1913.
[11] *Ibid.*, 461.
[12] *Ibid.*, 432.
[13] *Pub. Obs. U. of Michigan*, **3**, 106, 1919.

sented by the formula

Maximum of *Eu* II
= JD 2419869.720 + 5.46939E.

The departures of 10 epochs between 1913 and 1932 are irregular and never exceed 0.1 day. This must be regarded as exceedingly satisfactory. Incidentally, all recent observations, including those at McDonald and at Mount Wilson, in 1941, are perfectly well represented by her formula. This disposes of a doubt recently expressed by Tai [14] who thought that for his observations of 1938–39 maximum line-strength occurred about 1.2 days later than predicted from Miss Farnsworth's formula. I cannot explain the discrepancy, but when I apply the formula to the dates given his Table I, I find values which, with the exception of the first, are exactly one day smaller than the ones printed. This suffices to bring his observations into accord with the other series.

Returning to the paper by Kiess, we notice that he observed minimum velocity of λ 4205 on June 14.7, 1913 = JD 2419933.7. This is exactly 64.0 days later than the epoch of Miss Farnsworth's formula. Twelve times $P = 5^d.46939$ is 65.6 days. Hence maximum intensity for *Eu* II occurred 1.6 days later than minimum velocity—in excellent agreement with Belopolsky.

Kiess gave many new identifications of lines and suggested the presence of *Gd*, *Dy*, *Y*, *La* and, especially, *Tb*. The former two were confirmed by Morgan and later by Tai; *La* is uncertain according to the latter who found only some 19 unconvincing coincidences among 41 strong lines of Miss Moore's table; *Y* is definitely present. But *Tb* is uncertain, again according to Tai and also to Morgan, who examined the proposed identifications carefully and found that almost all lines formerly attributed to *Tb* were explained by *Fe* II, *Ti* II, *Cr* II, etc. This, incidentally, removes an important conclusion by Kiess. He had thought that the *Tb* lines belong to group B.

Kiess favored the idea of Fowler: "granting that there is a periodic change in level of the vapors of europium and terbium, radial motions and pressure effects combined may account for the velocity shifts observed."

Without going into further details, we may mention that the Russian investigators concluded in 1927 that the variations in line intensities were similar to those of the Cepheids. Miss Anger [15] gave new curves of intensities, and concluded, in agreement with Belopolsky,[16] that the spectrum has many emission lines, also of variable intensity. It is extremely difficult to decide whether these lines are real. Belopolsky thought that there were absorption lines with bright edges and also isolated bright lines. Miss Anger gave a long list of the latter, but no identifications. There are no identifications in the paper by Belopolsky. Although I have also occasionally suspected faint emission line on a few plates, I now believe that they are only spaces between absorption lines. It is, however, entirely possible that the spectrum has changed since 1928 when Belopolsky thought that the emission lines were strong.

We next come to a study by Miss Goldena Farnsworth [17] of Yerkes single-prism plates on Process emulsion taken in 1931 by several observers. Miss Farnsworth measured a number of lines for radial velocity and estimated their intensities. Her velocity curve for λ 4205 agrees with that of Kiess, and minimum velocity again precedes maximum intensity by about one day. But she suspected that the "large displacement of λ 4205 at minimum (intensity) phase was due to a blend" and suggested that the blending was with *Mn* II 4205.41. The line at λ 4205.48 is now believed to be due to *Fe* II. On Coudé plates this line is clearly resolved from the *Eu* II line, but with small dispersion the blend may easily be measured when *Eu* II is very weak. She also found a number of other lines in which blends of variable components unquestionaly produce spurious shifts in velocity. We shall return to this question later.

It is now appropriate to make an excursion into the field of photometry. In 1914 Guthnick and Prager found photoelectrically that α Can Ven is a continuous variable with an amplitude of 0.051 mag. The curve is roughly symmetrical. Maximum light occurred on 2420242.2 GMT, which is within 0.56 day of maximum *Eu* II intensity. All other reliable series of magnitudes by Guthnick and his associates, and most recently by Tai, agree in placing maximum light in coincidence with maximum *Eu* II intensity. But Tai, as well as the Babelsberg observers found that the range is not constant. Tai found 0.03 mag. Miss Güssow [18] records variations in shape and amplitude and a shift of − 1 day in

[14] *M.N.*, **100**, 94, 1939; *Observatory*, **63**, 134, 1940.
[15] *Ap.J.*, **70**, 114, 1929.
[16] *A.N.*, **234**, 5, 1928.
[17] *Ap.J.*, **75**, 364, 1932.
[18] *A.N.*, **237**, 334, 1930.

1928 and 1929 from the epoch of Guthnick. Sometimes the range is so small that the variation is not detectable. At other times it is fairly conspicuous. The period seems to be constant and agrees with that determined spectroscopically.

So far as I know, only C. T. Elvey has observed this star photometrically in the United States. He found variations in accordance with those of other observers. The results were never published. Tai's results were obtained in collaboration with H. E. Green at the photoelectric photometer of the 12-inch Sheepshanks refractor in Cambridge, England. The comparison star was δ UMa.

The color temperature was observed by D. R. Barber photographically at Sidmouth and the plates (14 in number) were measured by Tai. The results are regarded as somewhat uncertain because Hunter [19] had failed to find a variation within the period of 5.5 days in his work at Greenwich. If the curve is real it indicates a maximum temperature of 25,000° (or B5) near light minimum and a minimum of 18,000° (A0) near light maximum. Tai remarks, however, that the *He* I lines are always absent, and that the high temperatures are probably not real.

The latest spectroscopic studies are those by Morgan [20] and Tai.[21] Both contain excellent lists of lines and very full information concerning the identifications. Morgan's list is more complete, but Tai was able to use more recent laboratory data. He suggested, with a question mark, that *Ce* II might be present. The ultraviolet region was measured by me on Cassegrain plates taken at the McDonald Observatory,[22] but the phase of the plates corresponded to minimum strength of *Eu* II.

We now turn to the consideration of new data secured at the McDonald Observatory in 1941. The plates were obtained by Dr. P. Swings and by me with the Coudé spectrograph on seven successive nights, from January 21 to 27, 1941. They cover the region λ 3900 to λ 4700 and the dispersion is around 2 A/mm at the violet end and 4 A/mm at the red. The emulsion used was Eastman Ia-O, which is fast and very contrasty, but has bad grain. The spectra are greatly widened and this removes much of the difficulty usually caused by poor grain. It is surprising that we gain little when we used fine grain emulsions in our work with the Coudé. There are nine plates in all.

These have now been supplemented by nine more Coudé plates of exquisite quality from the Mount Wilson Observatory. We are exceedingly grateful to Dr. W. S. Adams for his generous offer to let us use his material in addition to our own. Finally, there are several visual Coudé plates by Swings, Henyey and myself, and also a good series of plates with the quartz Cassegrain spectrograph. The dispersion of the latter—all taken on Process emulsion—is 20 A/mm at λ 3250. The work here has only recently been started but I can give a few preliminary results.

In the light of the review of available literature, which I have presented, we can now specify our problem. We have been given ample evidence that α Can Ven is a very peculiar star. There are some others like it—mostly discovered at Yerkes Observatory by Mr. Morgan—but concerning these other stars we know even less. An attempt must obviously be made to study in detail those which have already been discovered, for example, 73 Draconis, BD $-$ 18°3789, θ Aquilae, ϵ Ursae Majoris,[23] and others; at the same time a fruitful field of endeavor would consist in the discovery of more objects having variable lines and in determining their periods. We may not be able to solve the mystery of these stars completely until we have enough material to apply statistical considerations. But that must wait until some one has the ability, skill and interest to undertake this work.

At present we shall have to limit our search for important new facts upon which to build a theory of those peculiar A stars to α Can Ven, their classical representative. In this connection we are quite naturally led to ask the following questions:

1. Granted that the rare-earth element europium has been identified with strong, variable lines in α Can Ven, what is the degree of certainty with which we have identified other rare earths? Kiess thought he had found *Tb* II, which was refuted by Morgan. Morgan says that *Dy* II and *Gd* II are quite strong, but Tai specifies his confirmation of *Dy* II with the words "seems to be unquestionable." He thinks that *Ce* II, *Tb* II and *Sa* II are "probably pres-

[19] *Observatory*, **62**, 219, 1939.
[20] *Op. cit.*
[21] *Op. cit.*
[22] *Contr. McDonald Obs.*, No. 13, 1939.

[23] Perhaps the most remarkable of these stars is ϵ UMa. Variable line intensities, especially of *Ca* II K, were discovered by Guthnick (*Sitz. Ber. Preuss. Akad. Wiss.*, 1931, p. 618; 1934, p. 13). The period is 5.0887 days.

ent," while for the other rare earths the evidence is described as insufficient. In another place he says that "the presence of *Sa* II . . . must be regarded as rather doubtful." It is obvious that on plates of small dispersion the positive identification of these and other rare earths will always remain difficult or even impossible. The high dispersion of the Coudé lends itself admirably to the solution of this question. Of course we must also look for other elements—not only the rare earths.

2. We next inquire: Do the radial velocities of the *Eu* II lines, as well as of other lines, really change, or are the variations reported by the early measurers solely due to blends, as Miss G. Farnsworth had thought?

3. Are there several groups of lines of variable velocity corresponding to the three intensity groups A, B and C?

4. Do all rare earths belong to group A, as Tai suggests by implication and, if so, are their variations alike in range and phase? How do the other elements vary?

5. Are the measurements of equivalent widths (E. W.) by Tai sufficiently reliable to serve as a basis for the computation of ionization phenomena, such as he has carried out?

6. Can normal changes in ionization, as postulated by Tai, account for the observational data?

7. Can we deduce anything from the fact that the lines of rare-earth elements are stronger than we had thought likely for an A-type star? Does this necessarily mean an exceptionally high abundance, as Tai has stated?

8. Does the high dispersion of our plates help us to find something significant in regard to the contours of the lines?

9. How does all of this tie in with the interpretation of the changes in α Can Ven and what are we to make of the hypotheses advanced by former workers?

10. Finally, what can we deduce from those peculiarities of α Can Ven, such as the weakness of *Ca* II, which it shares with many other A-type stars?

Our answers are preliminary in character, and may have to be revised after the work has been completed. But they will help us in trying to understand the nature of α Can Ven.

1. We start with the identifications. This part of the work is now under way and I can give some of the results. I have measured somewhat more than 1,000 lines between λ 3920 and λ 4670. There are some 650 lines in Swings' measurements between λ 3000 and λ 3930. I shall probably add some 400 new lines from the ultraviolet Coudé plates from Mount Wilson. Perhaps some 500 lines will come from the visual region. In all we shall have 2,500 to 3,000 lines—quite a catch for an A-type star. In α Cygni we measured less than one-half this number.

My present identifications are all based upon the measurements of the McDonald plates. We note at once that *Cr* II and *Mn* II are relatively very strong, while *Fe* II, *Ti* II, and *Sr* II are average and *Sc* II and *Ni* II are very faint. *Si* II is exceptionally strong, and *Ca* II is exceptionally weak. When I use the word "relatively" I mean that the intensities are compared with some sort of standard of relative intensities of these elements as observed in "normal" A stars, such as α Cygni, α Lyrae, or α Can Maj.

Let us write these elements in order of increasing relative strength and add the corresponding ionization potentials and excitation potentials:

Element	I.P.	E.P.
Ca II	11.8 *V*	0 *V*
Ni II	18.2	3
Sc II	12.8	0–0.3
Fe I	7.8	0–2
Fe II	16.5	2–3
Ti II	13.6	1
Sr II	11.0	0
Mn II	15.7	2
Cr II	16.6	2–3
Si II	16.3	10

There is no correlation with either ionization or excitation potential. The plate to which I refer is one at which *Eu* II is strong, but that makes no difference because these elements all vary little and the order would remain the same if we had chosen another phase.

But most of these lines belong to the groups of lines which vary little or not at all. We are more interested in the unusual elements, such as the rare earths. The evidence is exceedingly interesting. The wave lengths are, of course, accurate to within one or two hundredths of an angstrom for sharp, single lines. For faint lines the errors are larger, but rarely exceed 0.05 Å except when there are blends. Moreover, we have at our disposal enormously more lines than any of the previous workers. Tai used some 270 lines.

I find many new lines of *Eu* II; the limit is around laboratory intensity 15, when 5,000 is the laboratory intensity of λ 4205 and 5 or 6 its stellar intensity near maximum. There are virtually hundreds of lines of *Gd* II and especially *Ce* II.

Dy II is certain with numerous coincidences; so is *Nd* II, which is completely new. *Sa* II is certain, also with numerous unblended lines. *Ho* II is represented by only one line. But it is almost certainly real. I have not carefully examined *La* II, but I think it is also present. There has been no time to examine *Tb* II, *Pr* II and the less satisfactorily known elements *Er* II, *Tm* II, *Yb* II, *Lu* II. The pertinent data (which include those of Tai) for the rare earths are shown in Table I.

TABLE I

Element	At. No.	Atomic Wt.	First Ion. Potential	Second Ion. Potential	Presence in α Can Ven
Lanthanum	57	138.9	5.6 *V*	11.5	Present
Cerium	58	140.1	6.5	—	Present
Praseodymium	59	140.9	5.8	—	?
Neodymium	60	144.3	6.3	—	Present
Illinium	61	—	—	—	?
Samarium	62	150.4	6.5	11.4	Present
Europium	63	152.0	5.6	11.2	Present
Gadolinium	64	157.3	6.2	—	Present
Terbium	65	159.2	6.7	—	Probably present
Dysprosium	66	162.5	6.8	—	Present
Holmium	67	163.5	—	—	Probably present
Erbium	68	167.6	—	—	?
Thulium	69	169.4	—	—	?
Ytterbium	70	173.5	7.1	—	?

The rare earths account for a large number of lines. But there remain several hundred unidentified lines, some of them strong. This is undoubtedly the largest, if not the most important, unexplored domain of stellar spectroscopy.

2. We proceed with the next question, namely, that of the radial velocities. I want to subdivide this question into three sections:

(*a*) Do the normal, strong group-C lines show velocity changes?

(*b*) Do the *Eu* II lines change in velocity?

(*c*) Is the velocity of *Eu* II when at maximum equal to that of the normal strong group C lines? To answer (*a*), I have accurately measured three lines: *Si* II 4128, *Si* II 4131, and *Fe* II 4233. The results are summarized in the following list:

Phase	No. of Plates		Mean Velocity of Strong *Si* II and *Fe* II Lines	
−0^{d}04	1		+1.7	
+0.50	1		+0.7	+1.1 km/sec
+0.98	1		+0.2	
+1.48	2		+1.8	
+2.58	1		−0.3	
+3.51	2		−0.5	−0.3 km/sec
+4.52	1	(adopted)	0.0	

The total range is only 2.3 km/sec, which is probably not beyond the range of the possible errors of measurement or of the plates. There may be a suggestion that positive values predominate in the first half and negative in the second. It is safer, for the present, to suppose that these lines are stationary.

Since we probably can no longer believe that the lines of different groups come from different stars, we must suppose that the star is not a binary, but has a constant velocity. We turn to section (*b*). I have chosen *Eu* II 4205 and measured it accurately at those phases at which I can be certain that blends are entirely negligible. This precaution eliminates the doubt expressed by Miss Farnsworth.

Phase	Velocity of *Eu* II 4205	
−0.04	+9.0	
+0.50	+14.6	
+0.98	+22.4	
+1.48	+28.4	
+2.58	too faint	
+3.51	too faint	
+4.52	0.0	(adopted)

The progression is very definite and places beyond any doubt the reality of the smoothly ascending curve, beginning one day before maximum intensity of *Eu* II and ending near phase 1.48 days, where the line becomes too weak to be measured. I have plotted the results on Kiess' curve, assuming a zero point at phase 4^d.52 which agrees with his velocity at minimum. My measures were relative and give no absolute values. The agreement is perfect.

Finally, we test section (*c*). I have measured only one plate completely, namely, that at phase 4.52 days. We select four lines, two of *Fe* II, two of *Si* II, and two of *Eu* II.

Line	Group	λ Measured	λ Laboratory	Δλ	Mean Δλ
Fe II	B	4205.262	0.480	−0.218	−0.222
Fe II	B	4233.942	0.168	−0.226	
Si II	C–B	4127.809	0.049	−0.240	−0.266
Si II	C–B	4130.590	0.882	−0.292	
Eu II	A	4204.639	0.046	−0.407	−0.366
Eu II	A	4129.412	0.737	−0.325	

The difference

Eu II − *Fe* II = − 0.144 or − 10.2 km/sec
Eu II − *Si* II = − 0.100 or − 7.1 km/sec.

The difference is definite and real, but is appreciably smaller than that found by Kiess (− 17 km/sec) or even by Belopolsky, who finds:

Fe, *Mg* − 0.3 km/sec
Si II − 5.8 km/sec
Eu II at minimum velocity − 18 km/sec.

It is true, however, that my plate was taken at phase − 0.95 day counted from maximum of *Eu* II intensity, while minimum velocity on Kiess' curve falls 1.6 days earlier than maximum intensity.

We conclude that *Eu* II has a negative displacement before phase zero and a positive after phase zero. This may be due to real motion or to lack of symmetry of the contours. The plates themselves show no such asymmetry in λ 4129, which is free of blends. It is probable that the velocities represent real currents within a stationary atmosphere.

Miss Farnsworth's conclusions in regard to blending may apply to certain lines, but are not true of the strong lines of *Eu* II. It is, however, not clear why her velocity curve for λ 4205 ranges from 0 to about 30 km/sec, while that of Kiess goes from − 20 to + 15 km/sec. Slow changes may be present, but it is perhaps also possible that some error in the adopted procedure displaced her curve.

3. This question has already been answered in the affirmative. Whether all rare earths or, say, all lines of group A share the behavior of *Eu* II, I am not yet prepared to say.

4. I believe we can state from the preliminary identifications that the rare earths do, generally, belong to group A. But the range of variation in intensity is not the same for all elements. *Eu* II, *Nd* II and *Dy* II lead (A! in my designation), while *Gd* II, *Sa* II and *Ce* II have smaller ranges (A or sometimes C, especially for the fainter lines). Otherwise, there are also conspicuous differences in the range of variation. *Cr* II is conspicuously B! while *Fe* II is B and *Si* II is either C or B. In general, my results agree with those of Tai. *Ti* II is slightly A, *Ca* K is almost constant, as is *Mg* II 4481. *Fe* I is slightly, but definitely, of group A. There can be no doubt that the rare earths distinguish themselves only by the fact that *Eu* II, *Dy* II and *Nd* II are most conspicuously A!, but the tendency to change in phase with these elements is shared by others which are not rare earths.

5. We have here only a few preliminary data from one plate near *Eu* max. (phase − 0.09 day) and two near *Eu* minimum (phases 1.5 and 3.5 days). The results for *Eu* II 4129 are

Phase	No.	Equivalent Width
0 day.....	(1)	0.15 Å
2 days.....	(2)	0.02 Å

At minimum the line is invisible, and the ratio S of the equivalent widths (in Tai's designation) is at least 10. He uses 3 in his computations of ionization. Moreover Tai found at maximum 0.3 Å and at minimum 0.1 Å for λ 4129, or 2 and 5 times the values found here.

6. The procedure adopted by Tai consisted of the following steps:

(*a*) He attributed the great strength of the rare earths at maximum to unusually large abundance.

(*b*) He noticed that, in general, there appeared to be a tendency for elements of low ionization potential to belong to group A and of high ionization potential to group B.

(*c*) He chose *Eu* II for group A and *Ti* II and *Fe* II for group B.

(*d*) He assumed that the observed ratio $S = \frac{\text{E.W. at max.}}{\text{E.W. at min.}} = \frac{1 - x_1}{1 - x_2} = 3$.

(*e*) He computed under various assumptions of pressure and temperature changes the corresponding values of S and compared them with the observed values S_0 for *Ti* II and *Fe* II, after applying corrections for excitation by the Boltzmann formula.

(*f*) He chose as best fitting a change in p by a factor of 2 and in T from 9,000° to 10,000°.

(*g*) He acknowledged the fact that for *H*, *Si* II, *Mg* II, *Ca* II, *Sr* II, *Zr* II, *Y* II the same change in conditions would assign the lines to the correct group, but would predict a larger change in intensity than that observed.

(*h*) He concluded "we can then explain the spectral change in terms of thermal ionization and excitation."

If this were so, the problem would be relatively simple. But can we really invoke thermal ionization? The crucial point lies in the behavior of other elements than those considered by Tai:

(*a*) The ionization potential of *Sr* II is 11.0 volts. That of *Ca* II is 11.8 volts and that of *Eu* II is 11.2 volts. The lines of all three elements are ultimate, arising from the ground level. Computation by means of the ionization equation shows that $S = \frac{1 - x_1}{1 - x_2}$ for *Sr* II must be even

greater than for europium. We found for the latter $S \geqslant 10$. Yet, the strontium lines change very little in intensity, if at all. The discrepancy between *Sr* II and *Eu* II is much more significant than the agreement between *Eu* II and *Ti* II. It completely rules out an interpretation of the phenomena of variation in terms of the simple theory of thermal ionization. This does not mean that the theory is incorrect, nor that the theory has no application in the case of α^2 Can Ven. But it does mean that the observed phenomena are influenced by factors which are not encompassed within the scope of Saha's equation.

(*b*) The fact that S is probably greater than 10 for *Eu* II, instead of 3, as assumed by Tai, necessitates much larger changes in pressure or temperature, or both, than Tai had proposed. It is extremely doubtful that such major changes, for example one in temperature of the order of 2,000°, could take place without producing a much greater variation in light than the 0.03 mag. actually observed. Roughly speaking, the light changes as the fourth power of the temperature. A change from 10,000° to 12,000° should give us a photometric range of

$$2.5 \log \left(\frac{12{,}000}{10{,}000} \right)^4 = 0.8 \text{ mag.}$$

with maximum light when *Eu* II is weakest. In reality, the range is 0.03 mag., and the maximum of light occurs when *Eu* II is strongest.

(*c*) It is highly doubtful whether the observed ratio of equivalent widths can be identified with the ratio $S = \dfrac{1 - x_1}{1 - x_2}$ of the numbers of singly-ionized atoms. We know that in nearly all stellar spectra the curve of growth is such that the equivalent widths increase much slower than the corresponding numbers of atoms. For all but the very faintest lines a good approximation is

$$\text{E.W.} \sim \sqrt{NHf}$$

and

$$\frac{(\text{E.W.})_1}{(\text{E.W.})_2} = \sqrt{S}.$$

This makes our lower limit $S = 100$ for *Eu* II, and the corresponding changes in the physical conditions are even more extreme. We conclude that our answer to question 6 is "No."

7. The "rare earths" are, of course, relatively rare elements, but their abundances are not so small as to exclude their presence in stellar spectra. The important factor is the large number of levels which distribute the energy among many lines. Among the A-stars the temperature is so high that the ionization is nearly complete. Tai computes for *Eu* II at $p = 10^{-4}$ atmos., $T = 10{,}000°$:

$$\frac{x_1}{1 - x_1} = 83,$$

so that most of the atoms are doubly-ionized. It is, indeed, a fact that *Eu* II normally occurs in types F and G and we may assume that it reaches its normal maximum in these types. The strong lines in α Can Ven and other peculiar A stars are definitely abnormal.[24] They may be due to

(*a*) high abundance,
(*b*) suppression of double ionization,
(*c*) a special mechanism of line production such as may be caused by stratification.

We exclude from consideration (*a*) because there is not much that can be said about it. We have already seen that the large range in variation of *Eu* II is not due to normal changes in the physical conditions. Unless we are willing to think in terms of creation and destruction of *Eu* atoms, we cannot account for the great intensity of these lines at their maximum. At minimum there is not much to be explained. I doubt that (*c*) can account for the variations. At any rate, the relative weakness of those *Eu* II lines which lie in the ultraviolet and are within the wings of the Balmer lines suggests that the level of the *Eu* II lines is not unexpectedly high.

We are left with mechanism (*b*)—a variable degree of suppression of double ionization. Incidentally, various abnormalities caused by cycles within the *Eu* II spectrum are excluded because those lines which come from normal, not metastable, levels show corresponding intensities and variations. There is no appreciable geometrical dilution. Similar conclusions apply also to the other rare earths.

8. We find that the contours of the lines of groups C and B are somewhat broader, as if affected by greater turbulence or greater rotation, when the *Eu* II lines are weak. The significance of this result is not yet clear.

9. I believe that in the present stage of the work it is best to explore the hypothesis that small changes occur in the physical conditions which

[24] Morgan, *Ap.J.*, **75**, 53, 1932.

bring about large changes in the observed intensities. Such an effect is not possible when the physical conditions are similar to those of thermodynamic equilibrium. But we must remember that the Lyman lines and the Lyman continuum must be tremendously strong. The energy density which affects the ionization of the various elements is not that of a black body, and all kinds of departures must be important. Since all rare earths are of group A we are tempted to prefer an explanation in terms of abundance. But we must remember that the second ionization potentials of the rare earths are probably very similar. Hence it is not surprising that they behave in a somewhat similar manner. Whatever the agent may be that suppresses the second ionization of the rare earths at ionization potentials of 11.2 to 11.4 volts, it does not appreciably affect *Sr* II with ionization potential 11.0 volts or *Ca* II with ionization potential 11.8 volts. A strong absorption feature between 11.2 and 11.8 volts would perhaps answer the purpose, but in that case only a variation in the intensity of this feature could produce the variability of the observed line intensities. Perhaps there are other effects which can produce similar results. The motions of approach prior to intensity maximum and of descent following it suggest some type of selective radiation pressure with conversion of ionization energy into kinetic energy. The nature of the process remains obscure, but the fact that thermodynamic equilibrium is far from being applicable undoubtedly permits the mechanism to operate.

Incidentally, the *Eu* II line λ 4205 in δ Cephei seems also to be very sensitive to changes in intensity, and it certainly looks as though the phenomenon is similar to that observed in α^2 Can Ven.[25]

10. The proximity of the ionization energy of *Ca* II to L_β suggests that for this line the ionizing radiation may come, to an appreciable extent, from a bright chromospheric L_β line. This hypothesis would permit us to divide the A-type stars into two groups: those with pronounced chromospheres, in which *Ca* II is super-ionized, and those without chromospheres, in which *Ca* is under-ionized. This hypothesis has the advantage of shifting the responsibility for the variations in *Ca* II (as in ε UMa) to a chromospheric variation, which is *a priori* more probable than a large variation in pressure and temperature of the reversing layer. Swings and I have dealt with this hypothesis in another paper.

One other feature remains to be pointed out: the Balmer lines of *H* in α Can Ven are always appreciably weaker than in other stars of similar spectral type[26] and, according to Barbier, Chalonge and their collaborators, the Balmer jump in the continuous spectrum is nearly one-half of that observed in normal A0 stars.[27] But the Balmer lines are broad and the Stark effect is as conspicuous as in normal main-sequence stars. Low abundance of *H* may be the explanation, but it is difficult to see how this could reduce the intensity of *Ca* II and increase that of the rare earths. The *H* lines belong to group *B*, and are, therefore, weakest when the *Eu* II lines are strongest. But the change in equivalent width for *H* is not nearly as large as for *Eu* II.

It would be too optimistic to say that Cor Caroli has finally given up its secret. But I think we have accomplished two things: we have removed the problem from the list of astronomical mysteries, tinged by the superstition that Cor Caroli stands alone among the stars and by the naive belief that some sort of almost supernatural phenomenon gives rise to the creation and destruction of atoms of rare earths in a period of 5.5 days. On the other hand, we have been led to conclude that the other extreme of previous explanations, namely, in terms of the simple theory of thermal ionization, is equally dangerous because it obscures the most interesting and exciting characteristics of the star. We have suggested how the phenomena might be explained, and in doing so we have shown the way by which we shall travel toward our goal, the interpretation of the spectroscopic anomalies of α Can Ven.

[25] Krieger, *Ap.J.*, **79**, 98, 1934.

[26] See especially Plate XVI, *Ap.J.*, **90**, 1939.

[27] *Ann. d'Ap.*, **2** and **4**, 1938.

A COMPARATIVE STUDY OF GRAHAMELLAE, HAEMOBARTONELLAE AND EPERYTHROZOA IN SMALL MAMMALS

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I. INTRODUCTION

IN a previous study of the blood infections of the field vole, *Microtus pennsylvanicus pennsylvanicus*, attention was called to certain characteristics of the bartonellae of small mammals, differentiating them from *Bartonella bacilliformis* of the human being, and the genus *Haemobartonella* was created to include them (Tyzzer and Weinman, 1939). A question was raised as to whether the haemobartonella of the vole might assume a grahamella-like distribution in relation to the red blood corpuscles during certain phases of infection, not only in its natural host, but also in white mice inoculated with blood of infected voles. Subsequently, it was felt that certain conclusions in regard to the purity of the vole infection studied were open to question, and on this account a further investigation of the subject has been undertaken. The evidence now at hand indicates that we were dealing with a mixed infection and that the occurrence of rods distributed within the red blood corpuscles was due to the presence of a species of grahamella. The present study has, furthermore, brought out certain additional features of interest in regard to grahamellae, haemobartonellae, and eperythrozoa.

Primary Classification

The three types of organism under consideration may be distinguished on the basis of mor-

phology, their relation to the red blood corpuscles, reaction to splenectomy, reaction to arsphenamine, and ease of cultivation, these distinctions having been made through the combined contributions of various authors. Grahamellae as found in the blood present a marked degree of uniformity, appearing as rods with only an occasional rounded form, and are situated definitely within the red blood cells. They are not greatly increased by splenectomy, infect only a small percentage of the red cells, are not known to be pathogenic, are not eradicated by treatment with arsphenamine and are readily grown on certain media (Tyzzer, 1941). Organisms of this type stain readily and more intensely than some of the haemobartonellae.

The haemobartonellae exhibit great polymorphism, are distributed over the surfaces of the red blood corpuscles, are greatly increased by splenectomy and are often pathogenic for the splenectomized animal through the production of anemia. Organisms of this type differ from those of the genus *Bartonella* in that they show no evidence of multiplying in tissues other than the blood and do not produce cutaneous eruptions. They also may be eradicated by treatment with arsphenamine and, judging from the present study, are not readily cultured. Great differences are apparent in the haemobartonellae of various hosts in regard to their affinity for stains.

The eperythrozoa present a variety of forms, usually with a large proportion of ring-like elements, and occur on the surface of the red cells and in the blood plasma. They react to splenectomy and to arsphenamine as do the haemobartonellae and have not been grown in culture.

The criteria for the recognition of species of the genera, *Grahamella* Brumpt, 1911, *Haemobartonella* Tyzzer and Weinman, 1939, and *Eperythrozoön* Schilling, 1928, are rather meagre. The custom quite generally followed of differentiating species of organism primarily on the basis of host of origin implies a high degree of host specificity. From the ease with which certain of the haemobartonellae may be experimentally transferred to unrelated hosts, it seems at least possible that some such organisms may be encountered in more than a single host species in nature and also that more than one species of a type may be found in a single host species. However, there is at present no positive information in regard to either of these questions. Pathogenicity may increase during successive passages in a foreign host according to Adler (1930) who reported that *H. muris*, at first innocuous for non-splenectomized mice, later, after repeated passages became pathogenic. Hence it would appear that pathogenicity may not be regarded as a stable character. Other criteria to be considered, in addition to host of origin, are infectivity for various hosts, which may be taken as a measure of host specificity, morphology, pathogenicity, as indicated by blood changes and symptoms, and cultural characteristics if grown in culture.

There is in general no clearly distinctive morphology in the grahamellae of the various hosts studied. Ease of cultivation on artificial media not only sets the grahamellae off from the haemobartonellae but differences in growth characteristics support the differentiation of species, heretofore based on host of origin. The haemobartonellae often show no great degree of host specificity and are not readily propagated in culture so that—aside from the host of origin—morphology and infectivity or pathogenicity for various hosts are about the only criteria for the differentiation of species. In eperythrozoa, host of origin, infectivity for different hosts and morphology appear to be at present the only characteristics upon which species may be based.

Material and Methods

Infections of the three types of organism under discussion are found in trapped native mammals of various species. Grahamella infections are often apparent in non-splenectomized animals on examination of Giemsa-stained blood films. Splenectomy usually serves for demonstrating the presence of haemobartonella and eperythrozoön infections, and while the failure of splenectomy to make these infections apparent in the trapped animal furnishes grounds for assuming it to be non-infected, negative findings in regard to grahamella are much less conclusive. Rare infected red cells, found on a given date, may not be again demonstrable over a period of many weeks. The susceptibility of trapped animals without any demonstrated grahamellae in the blood to experimentally induced infection furnishes strong evidence as to their previous freedom from grahamella infection, while non-susceptibility may be based on immunity resulting from previous infection. The employment of laboratory-reared animals in experimental work such as that undertaken presents obvious

TABLE 1

INCIDENCE OF BLOOD PARASITES IN THE BLOOD OF SMALL MAMMALS FOLLOWING SPLENECTOMY

Animal*		Trapped†	Haemobartonella	Grahamella	Eperythrozoön
Vole	9714	Dec. 1939, Wakefield	0	0	0
	9812	Sept. 1940, Wakefield	+	0	0
	9825	Oct. 1940, Wakefield	+	+	0
	9826	Oct. 1940, Wakefield	+	+	0
	9827	Oct. 1940, Wakefield	+	+	0
Local Deer Mouse	9367	Aug. 1936, Martha's Vineyard	+	+	0
	9369**	Aug. 1936, Martha's Vineyard		0	
	9675	Dec. 1939, Wakefield	0	0	0
	9680	Dec. 1939, Wakefield	0	0	0
	9682	Dec. 1939, Wakefield	+	+	0
	9683	Dec. 1939, Wakefield	+‡	+	0
	9699	Dec. 1939, Wakefield	0	+	0
	9700	Dec. 1939, Wakefield	0	0	0
	9701	Dec. 1939, Wakefield	0	0	0
	9702	Dec. 1939, Wakefield	0	0	0
	9764	Feb. 1940, Westboro	+	+	0
	9765	Feb. 1940, Westboro	+	+	0
	9766	May 1940, Wakefield	+	+	0
	9767**	May 1940, Wakefield		+	
Gray-backed Deer Mouse	9817	Sept. 1940, Ossipee, N. H.	+	0	+
	9818	Sept. 1940, Ossipee, N. H.	+	+	+
	9819	Sept. 1940, Ossipee, N. H.	+	0	+
Pine Mouse	9732**	Feb. 1940, Athens, Ga.		0	
	9736**	Mar. 1940, Athens, Ga.		0	
	9738	Mar. 1940, Athens, Ga.		+	
Red-backed Mouse	9829**	Oct. 1940, Harvard		0	
Mole	9729**	Feb. 1940, Athens, Ga.		0	
	9734**	Feb. 1940, Athens, Ga.		0	
Shrew	9815**	Sept. 1940, Wakefield		+	
	9816	Sept. 1940, Wakefield	+	+	0
	9880	Feb. 19, 1941, Wakefield	+	0	
Gray Squirrel	9667	Nov. 1939, Norwell	+	0	0
"Peruvian Mouse"	9719	1936, Peru, S. A.	0	+	
	9720	1936, Peru, S. A.		+	
	9721	1936, Peru, S. A.		+	
	9722	1936, Peru, S. A.	0	0	

* Vole = *Microtus pennsylvanicus pennsylvanicus;* Local Deer Mouse = *Peromyscus leucopus novaboracensis;* Gray-backed Deer Mouse = *Peromyscus maniculatus gracilis;* Pine Mouse = *Pitymys pinetorum;* Red-backed Mouse = *Cleithryonomys (Evotomys) gapperi;* Mole = *Scalopus aquaticus howelli;* Shrew = *Blarina brevicauda (talpoides);* Gray Squirrel = *Sciurus carolinensis leucotis;* "Peruvian Mouse" = *Phyllotis darwini linatus.*

** Not splenectomized.

† Localities in Massachusetts, except when otherwise indicated.

‡ Infection became apparent 55 days after splenectomy.

advantages. Both voles and deer mice, reared under laboratory conditions with measures taken to eliminate ectoparasites, have proved to be free of all blood infections. The freedom of animals from haemobartonella and eperythrozoön has been checked by splenectomy, which has been done as a matter of routine, except for several animals injected with grahamella. For experi-

mental passage, a routine has been adopted of injecting non-infected animals both subcutaneously and intraperitoneally with the blood of an infected animal, suspended in normal saline or citrated saline solution. The elimination of certain organisms from a mixed infection may be accomplished in some cases by one or more passages in a foreign host, but this method for the isolation of a pure infection is not always reliable and should be carefully checked. Cultivation on artificial media is obviously advantageous, when possible. The accumulated experience in the cultivation of *Bartonella bacilliformis* has been utilized and many different media have been employed.

Morphological studies have been made in connection with each infection, blood films being prepared at frequent intervals and stained by the Giemsa method. Darkfield study has also been carried out, more especially of organisms grown in culture medium. There are complicating factors in the darkfield examination of infected blood and no new information has resulted from a limited number of observations.

The sources of the infections studied and some indication of their incidence are set forth in Table 1. Except in case of pine mice and moles and two grahamella-infected Peruvian mice in which only a single observation was made, repeated examinations of the blood were made at irregular intervals, an average of twelve blood films per animal being studied.

Most of the animals listed in the above table were splenectomized. In regard to those not splenectomized, we have no evidence as to the incidence of haemobartonella or of eperythrozoön, but such animals are included in order to indicate the incidence of grahamella.

For purposes of brevity, *Microtus pennsylvanicus pennsylvanicus* will be referred to in the text as the "vole," *Peromyscus leucopus novaboracensis* as the "local deer mouse" and *P. maniculatus gracilis* as the "gray-backed mouse." The semi-solid leptospira medium of Noguchi will be abbreviated to "leptospira medium." The question arises whether to follow precedent in describing the grahamella of each host as a separate species or to regard those of different hosts as adapted strains of a single species. The former alternative offers the advantage of providing a concise label for any future reference to a given organism. Hence, for purposes of identification and description, specific or varietal names will be applied to the organisms on the basis of host of origin, but on account of the limited information in regard to such organisms, the nomenclature here adopted may be regarded as tentative.

II. The *Grahamellae* of Various Small Animals, as Observed in Blood and in Cultures

Cultivation

In the course of attempts to isolate in culture the various haemobartonellae studied, the first definite growth was obtained in the semi-solid leptospira medium of Noguchi (Noguchi and Battistini, 1926), from a mixed grahamella and haemobartonella infection in the local deer mouse. This, at the time, was erroneously interpreted as the growth of the haemobartonella, was given only casual attention and not maintained. A second culture isolated from a similar mixed infection of another animal of the same species and carried through several passages in leptospira medium was employed four months later to inoculate two clean laboratory-reared deer mice which had been splenectomized. In one animal grahamella infection appeared in eight days, in the other, ten days after inoculation, and both remained free of any other infection. The organism was recovered in culture from the blood of both these inoculated animals two weeks after inoculation. With this evidence of the culturability of a grahamella, blood was taken from three trapped voles, each showing grahamella infection, and planted on various media. A pure culture of an organism growing in dense aggregates in semi-solid media was obtained from each. All three of these voles carried a latent bartonella infection as demonstrated subsequently by splenectomy. The inoculation of laboratory-reared voles with one of the culture strains was followed by grahamella infections. Subsequent splenectomy of the inoculated animals showed them to be free of any haemobartonella or eperythrozoön infection. An organism identical in character with that injected was isolated in culture from the blood of one of the inoculated voles.

An organism was grown in culture from the blood of a splenectomized gray-backed deer mouse showing a triple infection of grahamella, haemobartonella and eperythrozoön. This organism resembled rather closely the grahamella isolated in culture from the local deer mouse. The inoculation of two gray-backed mice and three local deer mice with culture material was followed by grahamella infection in all five ani-

mals. Three weeks after inoculation, blood from one of the local deer mice was planted on media and yielded a pure culture of the organism injected.

A short-tailed shrew, *Blarina brevicauda*, showing grahamella infection, also furnished growth on media inoculated with blood, which is probably that of the grahamella seen in the blood, but proof is lacking since no shrews known to be free of infection were available for test purposes. The above shrew after splenectomy developed a heavy haemobartonella infection. An organism was isolated in culture at this time identical with that obtained prior to splenectomy.

Thus, organisms isolated in culture from blood of two species of deer mice and of the common vole showing grahamella infections, have produced in turn grahamella infections when inoculated into other animals of the same species, and all three organisms have then been recovered in culture from the blood of the inoculated animals, thus fulfilling Koch's postulates. The employment of splenectomized animals for inoculation makes it reasonably certain that they carried no haemobartonella or eperythrozoön infection. It appears probable that the organism isolated from a mixed infection of grahamella and haemobartonella in the short-tailed shrew is also a grahamella, but in view of the frequency with which various types of organisms may be isolated from the blood of small mammals, this by no means should be assumed.

Grahamella peromysci, new species, of the Local Deer Mouse, *P. leucopus novaboracensis*

As seen in Giemsa-stained blood films, this grahamella possesses no features distinctive from grahamellae occurring in other host species examined.[1] In contrast to the variety of forms seen in the haemobartonellae, this organism shows rather uniform rods distributed within, rather than upon, the erythrocyte. Associated with the rods are occasional rounded coccoid forms which probably represent a developmental phase of the organism. The grahamellae are distributed throughout the red cell, characteristically lying apart from one another, the only exception being occasional paired rods interpreted as division forms. The grahamellae stain readily, often uniformly, but may show intensely stained granules. There is convincing evidence that the groups of extracellular grahamellae occasionally encountered in stained blood films are derived from infected red cells ruptured in the process of preparing the film.

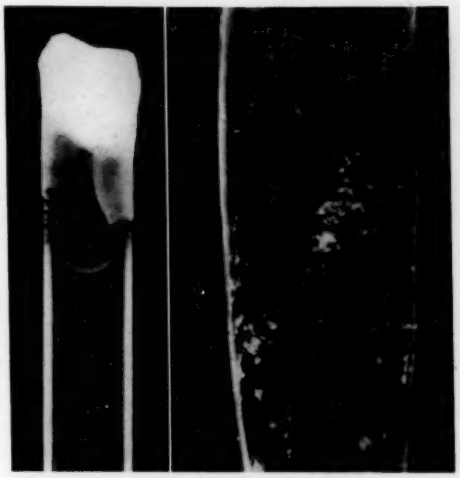


FIG. 1. Cultures of *Grahamella peromysci* of the local deer mouse. *A*. Growth in tryptose semi-solid medium. × 1. *B*. Growth on blood agar slant, 20 days, × 2.

Growth of this grahamella from blood planted in leptospira medium is first indicated by the development of minute, discrete, whitish particles appearing in from seven to ten days at 28° C., or after a longer interval at room temperature, 20–22° C. These particles increase in size and appear as irregular, sharply outlined white masses or flakes usually attaining their maximum development in a 5 to 10 mm zone [2] at the top of the medium. The initial flakes, evidently derived from a single infected red cell, never grow to more than 0.8 mm in diameter and from these dissemination occurs so that at a given time they may be surrounded by a cloud of satellite particles. With successive transfers of this organim, the tendency to particulate growth is largely lost and growth is indicated by the development of a zone of milky opacity at the top of the leptospira medium. By the inocula-

[1] In blood films of the European mole, showing *G. talpae* Brumpt, 1911, the type species of the genus, the organisms appear rather more intensely stained and of greater thickness than the grahamellae of local mammals, but since these preparations have been restained, it is possible that an excess of the dye has been deposited on the organisms.

[2] In order to photograph growths in semi-solid media, cultures of the various grahamellae were made in small flat tubes in which the characteristic subsurface zones did not develop. Allowance should be made for this in all the figures illustrating cultures in semi-solid media.

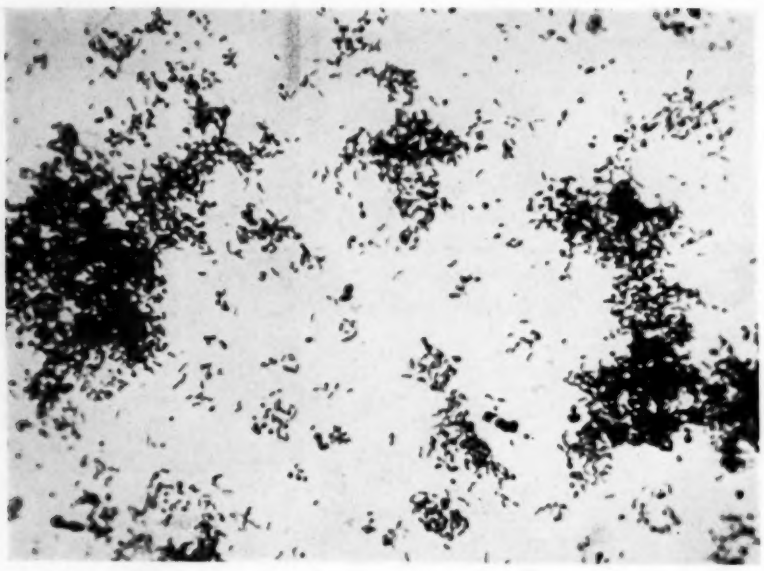


FIG. 2. Giemsa-stained film of *Grahamella peromysci* grown in semi-solid leptospira medium. Note rods, coccoid forms and larger globoid bodies. × 1275.

tion of this medium with relatively few organisms the particulate type of growth may be made to reappear (Fig. 1 *A*). This organism grows well on slants of certain solid media in case the latter is moist and provided with an abundance of fluid at the bottom of the tube. Only slight growth is obtained on NNN medium, but on blood agar having an enriched agar base prepared according to the specifications of the American Association of Medical Milk Commissions, Inc. (1937), abundant growth occurs. In sparsely sown slants individual colonies appear which are at first transparent, later opalescent, somewhat flattened, button-like, varying in size but rarely attaining a diameter of more than 1.5 mm (Fig. 1 *B*). In the course of ordinary transfers the organism grows over the surface of the slant in a continuous sheet having an uneven pebbly surface. This sheet appears at first rather moist and glistening but in the course of weeks becomes somewhat dull in appearance. Uniformly good growth has been obtained with blood agar made up with rabbit, horse and sheep blood. Growth is not quite so abundant on media containing 5 per cent blood as on media containing 20 per cent blood, but is, nevertheless, satisfactory. With blood agar slants a film of growth is often found extending for a time over the surface of the water of condensation, but this later falls to the bottom so that there is eventually an abundant, flaky deposit, which leaves the fluid wholly clear. Good growth is obtained on cystine blood agar and Bordet-Gengou medium when adequately moist. In peptone beef broth containing 2 per cent rabbit serum, *G. peromysci* forms a fine sediment at the bottom of the tube and the fluid becomes slightly clouded. In broth with hemoglobin, *i.e.* 2 per cent laked rabbit's corpuscles, the growth is more luxuriant and the fluid soon becomes clouded, while later a heavy precipitate forms and the medium assumes a dull brownish color. The organism, after becoming dispersed in fluid media, on transfer to leptospira medium, forms a whitish cloud without visible particles at the top of the medium.

G. peromysci, as grown on artificial media, presents a high degree of polymorphism and its morphology is obviously modified by the medium in which it is grown and the age of the culture. During early growth in leptospira medium the organism appears in compact clumps or aggregates of small elements, mostly rod-like in character (Fig. 2). Later the number of free organisms becomes progressively greater and coccoid forms and large globular bodies occur as well as rods. In media in which the organism occurs in aggregates, the peripheral portion of the organisms, as seen in Giemsa preparations, is tinted reddish and the outline is not distinct, while the central portion is deeply stained. Under conditions in

which the organisms are dispersed, as in old cultures and in peptone beef broth with hemoglobin, the organism assumes a more typical bacteriform aspect and presents sharply contoured rods and coccoid bodies, the former showing deeply stained granules often bipolar in position. During early cultivation there was scarcely any tendency to chain formation. After prolonged culture and on introduction to certain media, especially peptone beef broth with hemoglobin, the organism appears largely in chains or filaments with deeply stained granules at intervals (Fig. 13). The rods vary from 0.75 to 1.5 microns in length, rarely exceeding the latter figure. Such elements are slender, 0.25 of a micron in thickness when grown in leptospira medium and on blood agar, but when growing in chains in fluid media are from two to three times as thick. Coccoidal forms vary from 0.25 to 1.0 micron in diameter. Globular elements also appear as the culture ages. These vary in size up to 10 or 12 microns, stain unevenly, and are frequently bluish in lightly stained Giemsa preparations.

As already noted, the inoculation of two laboratory-reared deer mice of the local species with this grahamella culture resulted in grahamella infection (Fig. 3) and the organism was recovered in culture from both inoculated animals.

Grahamella peromysci maniculati, wen variety, of the Gray-backed Deer Mouse, *Peromyscus maniculatus gracilis*

This organism, as it occurs in the blood of an infected animal (Fig. 6), is indistinguishable from that occurring in the local deer mouse. In culture it closely resembles the grahamella isolated from the latter animal but its initial growth was rather more luxuriant. In the primary growth on leptospira medium the initial granules attained large size, reaching a maximum about 1 mm in diameter, while in subsequent transfers in the same medium, growth is indicated by a milky zone 4 to 5 mm in thickness at the top of the medium. Examination with a hand lens reveals that the milky cloud is constituted of minute particles (Fig. 4 *A* and *B*). On slants of blood agar made up with the enriched agar base alluded to above, the growth is rather more luxuriant than that of the grahamella of the local deer mouse, and appears as a sheet with an uneven, slightly wrinkled surface, or as moist, rounded, button-like, opalescent colonies (Fig. 4 *C*). Abundant flaky deposit develops in the water of condensation.

In fluid media, this organism grows more profusely in the peptone beef broth with hemoglobin than in the same broth with 2 per cent serum. It not only forms an abundant, finely granular

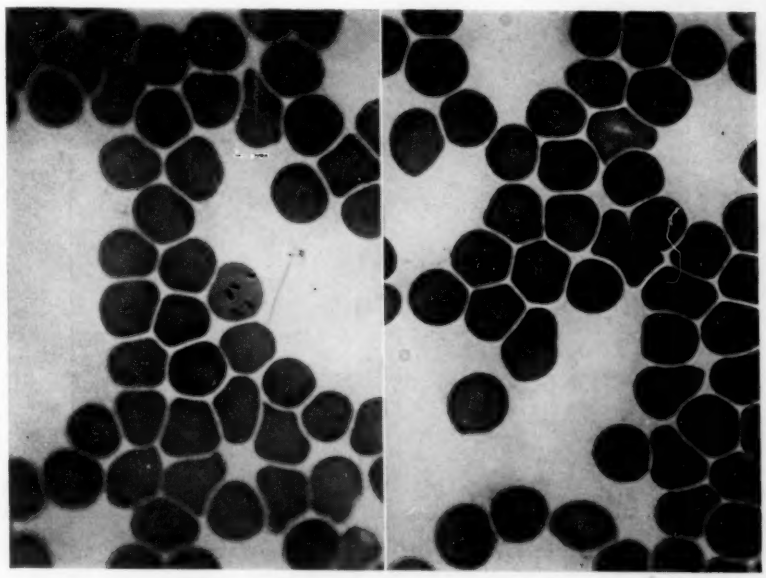


FIG. 3. Blood of laboratory-reared deer mouse of the local species, 11 days after inoculation of grahamella culture;—two red cells with typical grahamellae. Giemsa × 1275.

sediment, but is distributed in small colonies adherent to the wall of the tube, these being larger and more abundant toward the top.

In Giemsa-stained films from cultures in leptospira medium, the organism occurs in dense aggregates and free forms, the latter consisting of delicate rods with tapering ends, and small coccoids (Fig. 5). In fluid medium, it grows in chains, but on transfer back to leptospira medium, the organism no longer shows any marked chain formation. This strain of organism, unlike the one from the local deer mouse, has maintained some degree of particulate growth in fluid media and does not become so markedly bacilliform in its morphological variation. Rod forms from leptospira and blood agar cultures vary from 0.75 to 1.75 microns in length by 0.25 micron in thickness. The chains growing in fluid media may be about three times as thick and coccoidal forms vary from 0.25 up to 1.0 micron in diameter.

Two trapped, gray-backed mice whose blood had not shown grahamella were inoculated with the organism grown in culture, and developed an infection of the red cells which became apparent fourteen days after inoculation. (See Table 3.) Two splenectomized local deer mice were also injected, one with blood of a gray-backed mouse showing a triple infection of haemobartonella, eperythrozoön and grahamella, the other with blood of a gray-backed mouse at the height of an eperythrozoön infection. The first of the inoculated local deer mice developed a haemobartonellosis which alternated with an eperythrozoön infection, the other, a heavy eperythrozoön infection, but neither showed grahamella-infected red cells for seven weeks following the inoculation. Both animals were then injected with a culture of *G. peromysci maniculati* and showed infected red cells, one nine days, the other fourteen days later. A third laboratory-raised deer mouse of the local species was also experimentally infected by the injection on another occasion of a culture of the same grahamella, and showed infection twenty-two days after inoculation. Blood from one of the inoculated local deer mice planted on appropriate media gave growths of grahamella. Since this grahamella is infective for the local deer mouse, it may be regarded for the time being as a variety of *G. peromysci.*

Grahamella microti pennsylvanici, new variety,[3] of the Common Vole, *Microtus pennsylvanicus pennsylvanicus*

Grahamella appears to be more frequently demonstrable in voles approaching maturity than

[3] According to Yakimoff, 1929, *Grahamella microti* Lavier 1921 would appear to be preoccupied by *Grahamella arvalis* Tartakowsky 1913. However, the authenticity of Tartakowsky's species does not appear to be established. Yakimoff refers to the naming of the species in a catalogue of

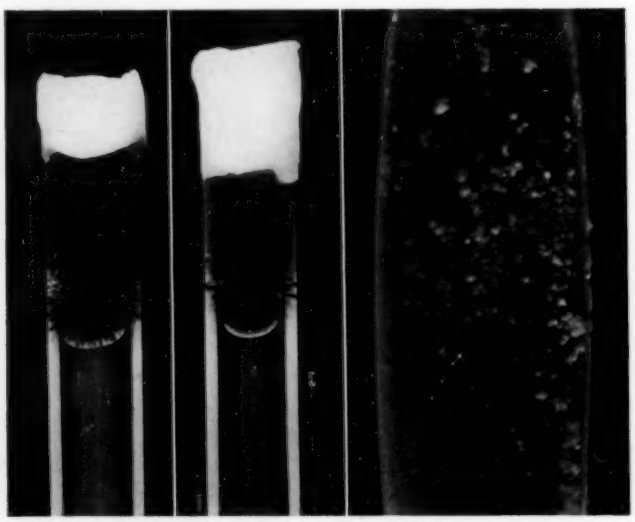


FIG. 4. Cultures of *Grahamella peromysci maniculati* of the gray-backed deer mouse. *A*. Leptospira semi-solid medium showing minute satellite granules spreading below coarser granules. × 1. *B*. Tryptose semi-solid medium × 1. *C*. Blood agar slant, 21 days. × 2.

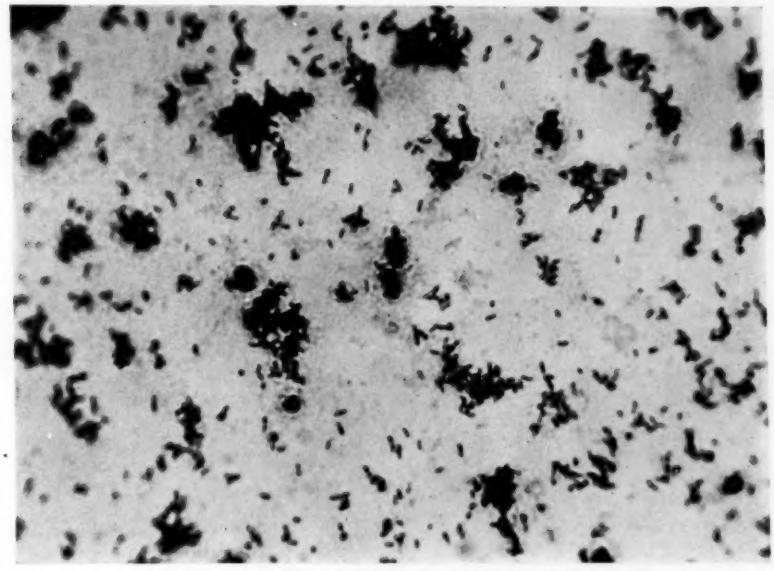


FIG. 5. Film from leptospira culture of *Grahamella peromysci maniculati*. Giemsa × 1275.

in old animals. The three culture strains isolated on one occasion from three trapped voles showing infected red cells appear to be identical, although only one has been tested out in compliance with Koch's postulates. Growth of the vole grahamella on various media shows one essential difference from the grahamella strains isolated from the two species of deer mice in its tendency to retain its particulate character in various media. In leptospira medium the organism grows in somewhat irregular, sharply outlined masses up to a millimeter in diameter, usually attaining greatest development in a layer up to 1 cm in thickness at the top of the medium (Fig. 7 *B*). In newly planted cultures particles may appear throughout the medium, but later the growth becomes more abundant toward the top and commonly constitutes an ill-defined zone of growth. On the blood agar slants growth appears in the form of definite colonies, never as a sheet (Fig. 7 *C*). With this grahamella the character of growth differs greatly with the content of the blood in the media irrespective of whether rabbit, sheep, or horse blood is used. While with 20 per cent blood growth continues and the colonies remain full and moist for a long period of time, with 5 per cent blood the growth of colonies proceeds for a relatively brief interval and is followed by regressive changes; the colonies become flattened, umbilicated, or umbonate, and have a dry, shriveled appearance. The regressing colonies commonly become iridescent and may present a peripheral zone of metallic, coppery-red color. The colonies of all three strains of the vole grahamella as grown on blood agar slants show great variation in type, not only in successive transfers but often in a single tube. The colonies may be classified as moist, translucent, conical; smooth, milky, low, button-like; and opaque, abruptly elevated, dimpled or granular. With such morphological differences, the possibilities of either the presence of more than a single species of organism in the culture or of dissociation within a single species were considered. With these questions in mind, single colonies of each type were picked and carried through a long series of transfers. No continuity of type was obtained but it was found that the type of colony depended upon the physical character of the inoculum, on the mode of inoculation, and on slight differences in the medium upon which it was planted. Thus, organisms

objects exhibited at an agricultural fair held in St. Petersburg in 1913. The validity of the species is contingent upon whether the name was accompanied by a description and whether it was effectively published, *i.e.*, printed or indelibly autographed and sold or distributed to the general public. (See International Rules of Botanical Nomenclature, Articles 36 and 37.) Since the catalogue in question is not at present available to the author, it is not possible to affirm Tartakowsky's species.

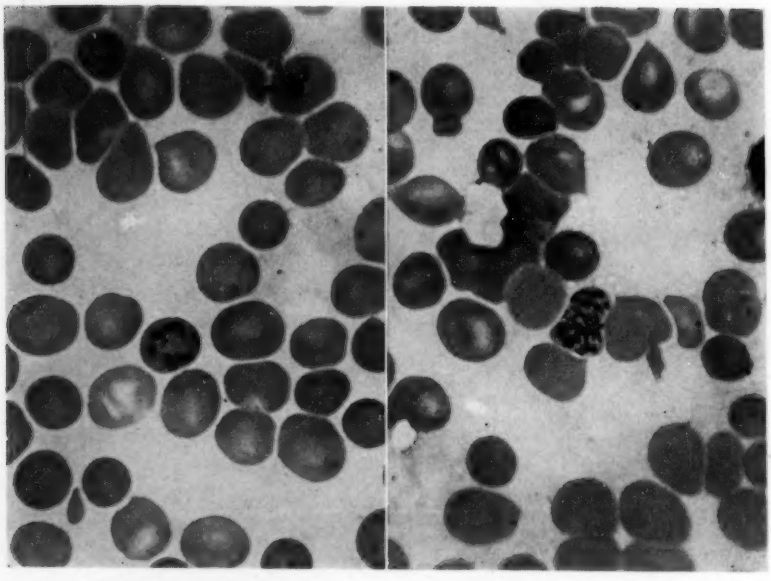


FIG. 6. Blood of splenectomized gray-backed deer mouse with mixed infection of grahamella and haemobartonella. *A*. Grahamellae within an infected erythrocyte and above and to right a group of grahamellae derived from a ruptured cell. *B*. A grahamella-infected red cell and above scattered rods and coccoid forms of haemobartonella. Giemsa × 1275.

dispersed in fluid produced moist colonies on a moist medium, but deposited with leptospira medium on the blood agar slant produced opalescent, button-like colonies. The deposit of large aggregates or fragments of colonies furnishes dry, granular, or dimpled masses, often growing to great size. All types of colony are non-adherent and slip over the surface of the slant at the merest touch. The inoculation of leptospira medium with all types of colony furnishes growths which are identical in every respect. Giemsa-stained films of the various types of colonies show only a single type of organism. Hence it would appear that we are dealing with pure cultures in all three strains derived from the vole.

G. microti pennsylvanici grows in such dense, tough colonies that considerable manipulation is required in the preparation of films to obtain sufficient free organisms for study (Fig. 8). In fragments of colonies seen in Giemsa-stained films, the organisms appear to lie in a matrix by which they are cemented together in compact masses. The organism may be either rod-like or coccoid in form, the relative proportions of the two varying. Thus, after two weeks' growth on blood agar, the colonies may consist almost wholly of coccoid and globular forms, some appearing hollow or ring-like. With darkfield illumination, a segment of a colony may present a rounded mass of coarse coccoid forms, while the remainder of the mass consists of rods. The latter average somewhat thicker than the rod forms of the deer mice. The rods show a range of variation, 0.7 to 2.7 microns in length, the thickness being quite constant, approximately 0.33 micron, and the coccoids vary from 0.33 to more than 1.0 micron in diameter. The rod forms commonly taper to the extremities, show a deeply stained central portion and a faintly stained peripheral portion, and are without a distinct contour. It is possible that the indistinct peripheral substance represents a secretion rather than an integral part of the organism, for under certain conditions this material is not apparent and the outline of the organism is well defined. In some preparations an excess of stringy mucoid material is encountered in which the organisms appear large and distorted as though flattened. The excess of external substance or matrix and the firm adherence in dense masses are outstanding features of the vole grahamella as grown in culture media.

Two young laboratory-reared voles with no organisms microscopically demonstrable in the blood were inoculated, one with organisms from an original culture twenty-three days after being

planted with blood, the other with organisms from the third transfer forty-seven days after isolation in culture. Both animals showed numbers of grahamella-infected red cells five days after inoculation, the cells at this stage of the infection showing relatively few organisms, *i.e.* from one to four per cell. Blood from the second vole taken ten days after its inoculation furnished characteristic cultures on both leptospira and blood agar slants. The susceptibility of white mice to the vole grahamella was tested. Series of both splenectomized and young normal white

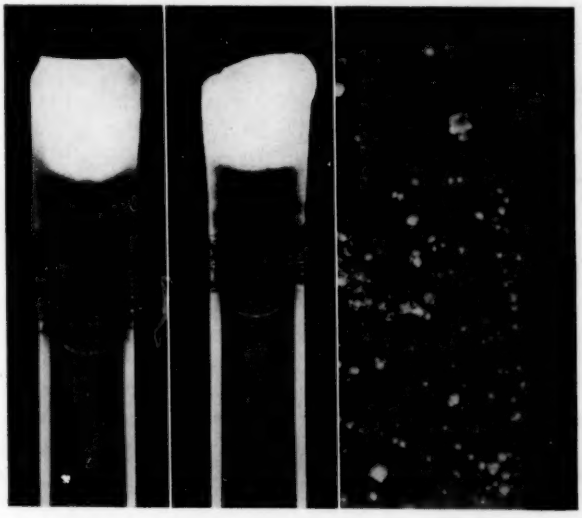


FIG. 7. Cultures of *Grahamella microti pennsylvanici* of the common vole. *A*. Leptospira semi-solid medium. × 1. *B*. Tryptose semi-solid medium. × 1. *C*. Blood agar slant 21 days. × 2.

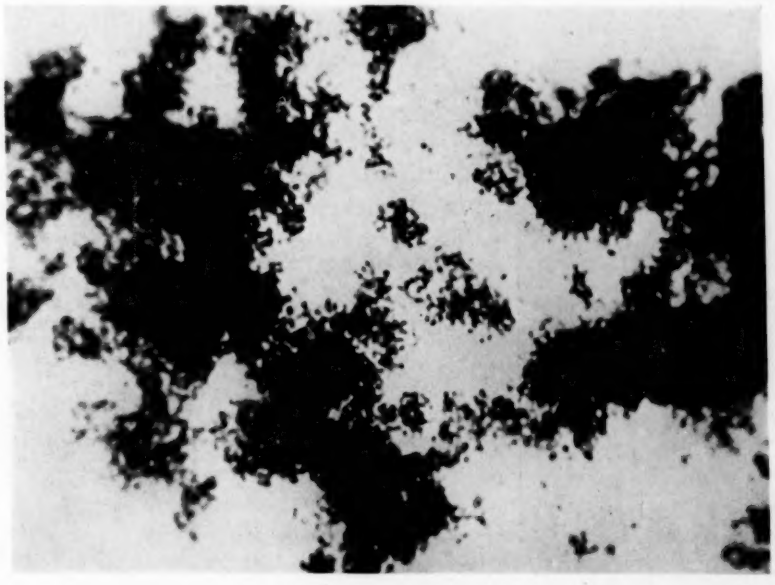


FIG. 8. Giemsa-stained film from grahamella culture derived from the blood of an infected vole. Note thickness of organisms and occurrence in compact masses. Semi-solid leptospira medium. × 1275.

mice were injected with culture material, some being injected also with haemobartonella-infected vole blood, but no animal furnished any microscopic evidence of grahamella infection. The haemobartonellosis which resulted in two splenectomized mice inoculated with both cultured grahamella and bartonella-infected vole blood was carried through four successive transfers in splenectomized white mice. Although grahamella was never seen in blood films, it was isolated in culture from one of the first mice inoculated, twenty-four days after its injection, from the two mice of the next transfer seven and eleven days respectively after their injection, but all attempts at isolation from the mice of subsequent transfers failed. The cultures isolated from the inoculated mice have not been tested by the inoculation of voles as no laboratory-reared stock of these animals is now available, but on comparison of the organisms isolated with the strain of grahamella injected into the mice, all present the same type of growth on various media. Thus, it seems clear that the grahamella of the vole may persist in white mice when not microscopically demonstrable. On the other hand, it appears that on occasion the vole grahamella may multiply sufficiently on injection into the splenectomized white mouse to be microscopically demonstrable in the blood for a brief time, as shown in earlier experiments (Tyzzer and Weinman, 1939).

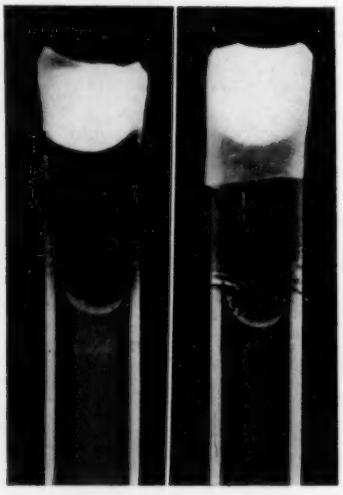


FIG. 9. Cultures derived from the blood of a short-tailed shrew, not tested by shrew inoculation. *A*. Leptospira semi-solid medium. × 1. *B*. Tryptose semi-solid medium. × 1.

Grahamella blarinae, new species, of the Short-tailed Shrew, *Blarina brevicauda*

The blood of two shrews, both taken in a trap set repeatedly in one locality, showed grahamella-infected red cells. One, kept for nearly three months in captivity, was splenectomized and subsequently developed a severe haemobartonellosis. Blood from the tail was taken both before and after splenectomy, and planted on various media. The same organism was isolated from blood taken on October 1 and that taken on November 6, splenectomy having been performed on November 1. While the organism grown (Figs. 9 and 10) resembled somewhat in its cultural characteristics the grahamellae isolated from the two species of deer mice, no laboratory-reared or demonstrably uninfected shrews have been available upon which to test the culture. The blood of the grahamella-infected shrew taken before splenectomy was used to inoculate two splenectomized deer mice, *P. leucopus novaboracensis*, and again after splenectomy, when the haemobartonellosis was at its height, to inoculate two splenectomized white mice. The subsequent examination of stained blood films of the inoculated animals taken over a period of several weeks gave no evidence of the transfer of infection. The injection of a young deer mouse of the above species with the organism grown in culture also resulted negatively. It was hoped that a third shrew trapped subsequently in a different locality might serve as a test animal, since its blood presented no grahamella-infected red cells. After being splenectomized with the possibility in mind of making apparent its latent infections, it developed a severe haemobartonellosis which was maintained throughout the period of nearly seven weeks for which it survived. No grahamellae were found in stained blood films taken at intervals throughout this period. Culture media planted with splenic tissue at the time of operation and a great variety of media planted ten days later with blood furnished no growth. On account of the unexpected death of this animal, the inoculation test for the organism previously isolated from shrew's blood was not carried out.

This organism in leptospira medium forms a milky cloud in a zone from 5 to 8 mm in thickness at the top of the medium. Examined with a hand lens, the zone shows minute, whitish particles in suspension, but there appears to be also a flocculation of the medium itself. On blood

FIG. 10. Giemsa-stained film of organisms isolated from blood of shrew;—from a colony on blood agar slant. × 1275.

agar the organism grows in a sheet having a coarse, pebbly surface as though composed of confluent colonies of various sizes.

The organism cultured appears in the form of delicate rods from 0.75 to 2.3 microns in length by about 0.3 to 0.4 micron in thickness, which usually taper slightly toward the extremities, and may be slightly bent (Fig. 10). In some preparations the rods stain uniformly, presenting no granules, but usually a considerable proportion of the rods show a bead-like swelling, more frequently but not always centrally situated. Coccoid forms from 0.3 to 1.0 micron in diameter, and even coarser globules, occur. The larger globules stain bluish in lightly stained preparation and their origin is quite obviously from the swelling of rods, every stage of the process being apparent. Aggregates occur which are for the most part composed of such globules with a relatively small proportion of rods and coccoids. Under certain conditions of growth, the rods and coccoids may be dispersed and, as seen thickly distributed in the film, have a bacterium-like morphology. Usually the preparations show loose aggregates as well as free forms. In leptospira medium and on blood agar there is only slight tendency to form chains. This organism grows more profusely at 37.5° C. on blood agar slants than the three proved strains of grahamella, the medium turning a dull chocolate color.

Grahamella phyllotidis, new species, of the Peruvian Mouse, *Phyllotis darwini linatus*

Two of four trapped Peruvian mice presented grahamella infections peculiar in that the infected erythrocytes contained an unusually large number of organisms, in some instances so densely packed that the infected cell was intensely stained and resembled a nucleus (Fig. 11). In an attempted splenectomy both infected animals reacted badly to ether anesthesia and succumbed. Splenectomy was successfully performed on the two animals without visible infection and for a period of eleven days following the operation the blood of both showed no grahamellae. They

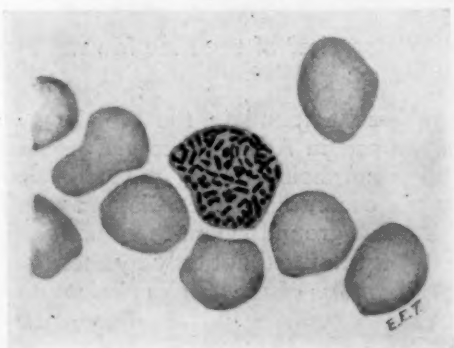


FIG. 11. *Grahamella phyllotidis*. Blood of Peruvian mouse, *Phyllotis darwini linatus*. Giemsa × 2000.

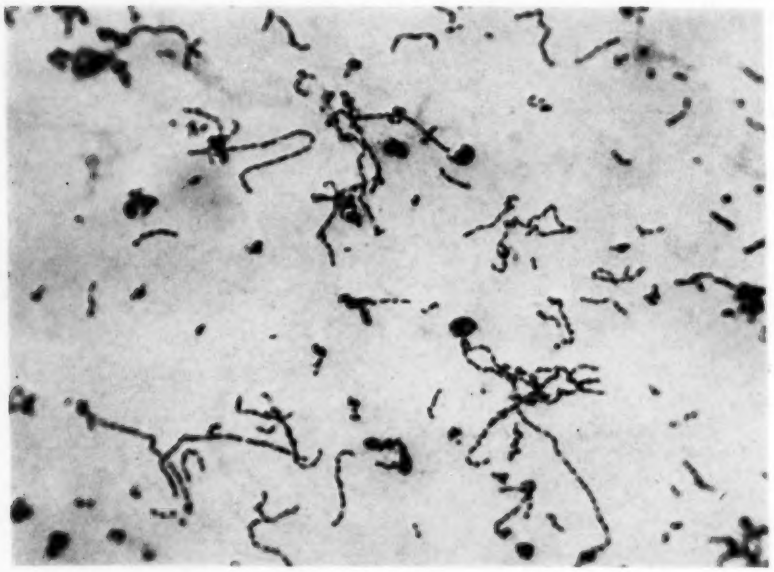


FIG. 12. Chain-forming organism isolated from blood of vole. Note large rounded bodies which appear early in development. Leptospira semi-solid medium. Giemsa × 1275.

were then injected with the blood of a local deer mouse showing grahamellae, and six days later one presented numerous infected red cells. Many of these were well packed with rods, as was the case in the naturally infected *Phyllotis*, so that it is probable that the infection was a natural rather than an induced one. Two attempts to recover the grahamella in culture, in one case using tail blood, in the other heart's blood, yielded pure cultures of an organism growing in beaded filaments and forming loose aggregates of rods, coccoids and coarse spheroids in semi-solid media. The *Phyllotis*, showing no grahamella, also yielded the same organism. No clean stock was available for inoculation tests.

Grahamella species? of the Pine Mouse, *Pitymys pinetorum*

A grahamella found in the blood of a pine mouse trapped in Athens, Georgia, presents no distinctive morphology, being quite similar to the grahamella occurring in the vole and deer mice studied. The ground where the pine mice were trapped was tunnelled not only by these animals but also by moles of the species *Scalopus aquaticus howelli*, so that in this case as with other closely associated species of mammals, the possible sharing of a single species of grahamella by different host species may be considered. The cultivation of the grahamella found in the pine mouse was not attempted. A splenectomized white mouse injected with blood of this pine mouse failed to develop a visible infection.

Other Organisms Isolated from Blood

In addition to the proved strains of grahamellae, several other possibly related organisms have been isolated from the circulating blood of the vole. Certain of the features of these organisms may be of sufficient interest to be presented in some detail. Two strains of organisms (9825 and 9827) isolated from two grahamella-infected voles simultaneously with the tested strain (9826) of vole grahamella are culturally identical with the latter and are regarded as grahamella strains. Later on cultures were obtained from the heart's blood of two voles trapped in cold, winter weather and found frozen. Another culture was obtained from the heart's blood of an old vole inoculated long previously with grahamella grown in culture. No grahamellae were demonstrable in post-mortem blood films prepared from the latter three animals at the time of planting of cultures, but this finding is probably of less significance than in case of blood films made during life. All three strains were Gram-negative and non-acid-fast.

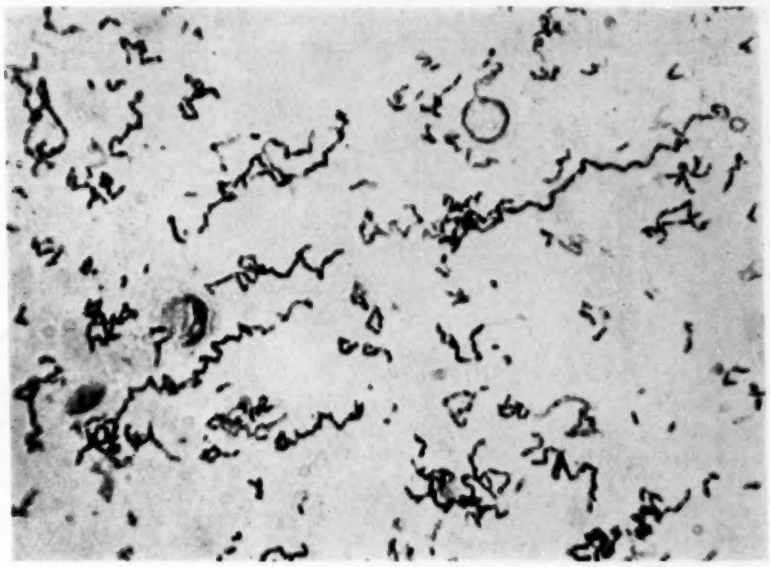


FIG. 13. Chain formation in *Grahamella peromysci* grown in peptone beef bouillon with hemoglobin. Giemsa × 1275.

Strain 9879.—Source, heart's blood of vole taken frozen from trap, Wakefield, February 20, 1941. A growth obtained in leptospira medium appeared as a fine granular cloud at top of medium. The granules were soft and easily spread, not in tough masses as in cultures of the vole grahamella. In subsequent transfers, a whitish cloud forms at the top of leptospira medium, the colonies on blood agar are similar to those of grahamella but tend to coalesce. In peptone beef broth with hemoglobin, growth seen in minute particles on walls of tube—fluid clear; in peptone beef broth with serum, similar but less abundant growth. Microscopically the cultures show an organism growing largely in filaments and forming loose aggregates. Individual rod forms are no longer but average thicker than those of the vole grahamella. Intermingled with the small elements are unevenly stained globules, bluish when lightly stained and often attaining a diameter of four microns. Films from blood agar colonies present rather dense masses, in the interior of which are large amounts of blue-stained amorphous material associated with recognizable organisms. Smaller aggregates consist of organisms intermingled with coarse globules. Non-pathogenic for the white mouse.

Strain 9888.—Source, heart's blood of vole taken dead from trap, Wakefield, March 26, 1941. Growth in leptospira medium in granules near top resembling those of grahamella. On blood agar colonies moist, opalescent, not becoming umbilicated as seen in vole grahamella colonies. In peptone beef broth with hemoglobin there is abundant growth in film on wall of tube, fluid not clouded, and in peptone beef broth with serum, growth in granules on wall of tube, no clouding—the growth in both fluid media being more abundant and quite unlike that of the proved grahamella culture. In stained films this organism occurs in tortuous beaded filaments showing large laterally-projecting elements suggesting branching. Free globoid or rounded elongate bodies occur in considerable number (Fig. 12). Some take a clear blue stain, others stain unevenly, often with a suggestion of included granules. Such globules occasionally attain the size of a leucocyte. Aggregates occur composed in part of coarse globular elements, in part of recognizable organisms. In films of older cultures there are dense masses of reddish ill-defined elements. This organism is probably slightly pathogenic, one of four white mice, injected subcutaneously and intraperitoneally, dying within four days.

Strain 9812.—Source, heart's blood of an old vole trapped about one year previously. Following splenectomy, September 26, 1940, a severe bartonellosis developed which remained apparent up to December 10, 1940. At no time was grahamella found in blood films. Following the injection of grahamella culture material on December 3, grahamella-infected red cells appeared in the blood on December 17. The animal when killed, April 21, 1941, appeared scrawny, showed defective teeth and swelling near the heel of one hind foot. Leptospira media planted with heart's blood in six days furnished a growth appearing as whitish granules near the top of the medium. Transferred to blood agar slants, growth appeared in small transparent colonies and milky coalesced colonies forming a thick elevated

growth resembling a thick film of paraffin. In fluid peptone beef broth and hemoglobin, a rather heavy sediment and fine particles adherent to wall of tube appear, the fluid remaining clear. Stained films show an organism growing in beaded filaments and compact masses. Great numbers of clear blue bodies, rounded or amoebiform, often attaining the dimensions of red blood corpuscles. Some preparations show aggregates composed largely of globoid bodies. No morbidity followed the subcutaneous and intraperitoneal injection of white mice.

In addition to the foregoing strains of organisms derived from the heart's blood of the vole, an organism which grows in chains or filaments has been isolated on one occasion from the tail blood of one and on another occasion from the heart's blood during life of two Peruvian mice of the species *Phyllotis darwini linatus*, one of which presented a grahamella infection. This organism, which has been already briefly discussed, grows more rapidly than the various cultured strains of grahamella, in leptospira medium producing a surface zone of loose fluffy growth, and on blood agar grows as a film or in distorted drop-like colonies, and produces a flocculent growth in the fluid media. It is Gram-negative, does not form compact aggregates, and is non-pathogenic for white mice. The deeply stained elements within the filament may appear as minute rods and much thicker coccoids, interspersed with bead-like elements which attain relatively great size. The stained elements of the filament are regularly spaced, the unstained interval being nearly equal to the length of the stained portion except in case of the larger elements. The morphology of this organism is strikingly similar to that of the one isolated from Haverhill fever (Parker and Hudson, 1926). Cultures of an organism which has been isolated with great regularity from this species of *Phyllotis* by Dr. Marshall Hertig, working in Peru, have been submitted for examination and agree in every respect with those of the organism just described.

To summarize—the planting of blood from a small series of trapped voles on appropriate media has yielded cultures of non-motile, Gram-negative organisms having a propensity to grow on certain media in beaded filaments and in each strain associated with coarse globoid bodies. Only one of these strains has proved slightly pathogenic for young white mice on intraperitoneal and subcutaneous injection. While there are also certain similarities in the gross appearance of the cultures and those of grahamella, the discrepancies are such as to indicate that they are distinct from the vole grahamella. Not only is the frequent occurrence of streptobacillus-like organisms in the blood of the vole and the Peruvian mouse of interest, but the strains obtained may prove useful for further comparative studies in determining the group relationships of the vole grahamella. Serological studies should throw light on the question of their antigenic relationships. (*Listerella monocytogenes* has also been isolated from the central nervous system of a vole, obviously abnormal when taken from the trap and later developing paralysis of the hind legs. Identification by Dr. LeRoy Fothergill.)

General Features and Systematic Position of Grahamellae

All strains cultivated are Gram-negative, non-acid-fast and non-motile. They are not well stained by ordinary aniline dyes, individual organisms being scarcely distinguishable after Loeffler staining, but are well stained by the Giemsa method. They grow readily at room temperature of about 20° to 22° C., also at 28° C. and 37.5° C., are aerophilic but will grow to some extent at the bottom of certain fluid media. They grow luxuriantly on solid media containing large amounts of hemoglobin and also in semi-solid media containing serum in which there is a trace of hemoglobin. Such media as nutrient agar or peptone beef broth do not support growth. The survival properties of all strains kept at room temperature are great, cultures of each strain having furnished growth on transfer after several months—*G. peromysci* after more than eleven months. The grahamellae cultivated from three different hosts have all shown under certain conditions some degree of propensity to grow in beaded unbranched filaments or chains and to produce coarse globoid bodies suggestive of forms occurring in *Actinomyces muris* [4] cultures.

In regard to classification, the possible relationship of organisms of the genera *Grahamella*, *Bartonella*, *Eperythrozoön* and *Anaplasma* have been discussed by various authors, Neitz, Alexander and du Toit, 1934, having suggested the

[4] *Streptobacillus moniliformis* Levaditi, Nicolau and Poincloux, 1925, is considered by Topley and Wilson, 1936, to be identical with *Haverhillia multiformis* Parker and Hudson, 1926, and with *Streptothrix muris ratti* Schottmüller, 1914, the latter being placed in the genus *Actinomyces* by Lieske, 1921. Topley and Wilson have suggested the binomial *Actinomyces muris*.

family Anaplasmidae, evidently as a zoölogical rather than a botanical group. Since through cultivation *Bartonella bacilliformis* is revealed to be a flagellated bacterium, and as the propagation on dead media of grahamellae from various hosts furnishes conclusive evidence as to their bacterial nature, the question arises as to whether to continue to consider the blood parasites enumerated above in a major group of uncertain systematic position, as previously suggested, or to undertake the introduction of those organisms which have become better known through cultivation into existing systems of classification. The latter alternative is preferred by the author. Although certain mycologists would place the actinomyces with the HYPOMYCETES-FUNGI IMPERFECTI (Dodge, 1935), bacteriologists find reason to include this type of organism in the SCHIZOMYCETES. In undertaking to classify the *Grahamellae*, the bacteriological system will be followed.

It seems clear from the present studies that organisms, heretofore known as grahamellae, show on cultivation characters suggesting close relationship to organisms of the *Actinomyces* (*Streptothrix*) *muris* type in regard to morphology, growth in filaments, associated globoid bodies, affinity for stains and general cultural characters. If Bergey's "Manual of Determinative Bacteriology," 1939, is followed, they should be included in the family ACTINOMYCETACEAE Chester, order ACTINOMYCETALES Buchanan, class SCHIZOMYCETES. Not only should the grahamella type of organism be incorporated into bacteriological taxonomy, but also it would appear important for the genus *Grahamella* to be retained, on account of peculiar features in regard to the favorable effect of hemoglobin on growth, as well as to habitat and peculiar limitation of multiplication in infections.

In case the classification set forth in Bergey's "Manual of Determinative Bacteriology," 1939, is followed, it is suggested that the key be modified in some way such as the following to include the genus *Grahamella:*

KEY TO THE GENERA OF FAMILY ACTINOMYCETACEAE

I. Rods or filaments, usually unbranched
  A. Unbranched, frequently clubbed, human mouth parasites
    Genus I. *Leptotrichia*
  B. Filaments frequently thickened, occasionally branched. Skin parasites of hogs, possibly of man
    Genus II. *Erysipelothrix*
  C. Primary growth generally in small mycelia later forming shorter rods and coccoid forms. Mostly soil forms
    Genus III. *Proactinomyces*
  D. Slight propensity to grow in unbranched filaments variable; rods and coccoids with indistinct contour commonly cemented together in dense masses; growth favored by hemoglobin; nonmotile, aerobic, Gram-negative, with slight affinity for ordinary aniline dyes, well stained by Giemsa; within red blood corpuscles, non-pathogenic
    Genus IV. *Grahamella*

Type species—

Grahamella talpae Brumpt, 1911
Red blood corpuscles of European mole, *Talpa europaea*

Species isolated in culture—

Grahamella peromysci, new species
Red blood corpuscles of *Peromyscus leucopus novaboracensis*. Becomes dispersed in semi-solid media, growth in sheets on surface of blood agar

Grahamella peromysci maniculati, new variety
Red blood corpuscles of *Peromyscus maniculatus gracilis;* infective also for *P. leucopus novaboracensis*. Growth in culture luxuriant; forms colonies in fluid media

Grahamella microti pennsylvanici, new variety. (*Grahamella microti* Lavier 1921)
Red blood corpuscles of *Microtus pennsylvanicus pennsylvanicus*. Tends not to become dispersed in growth in culture, cemented into tough aggregates, on solid media grows in discrete colonies rather than in sheets

It would appear, however, that there is no general agreement among bacteriologists in regard to classification of organisms included in the order ACTINOMYCETALES. The subject has been recently reviewed by Waksman, 1940, who offers a classification with the family PROACTINOMYCETACEAE, including only mycelium-producing organisms with spores formed by segmentation. No discussion is given of *Actinomyces* (*Streptothrix*) *muris* type of organisms; hence it is not known whether they would be included in the family MYCOBACTERIACEAE, which is distinguished by "mycelium, rudimentary or absent." It is obvious that organisms of the grahamella type may not be properly included in either *Mycobacterium* or *Corynebacterium*, the only two genera listed by Waksman in the MYCOBACTERIACEAE. The classification which he proposes is thus not wholly satisfactory in that his

groups are not sufficiently comprehensive as to include many closely related organisms.

It is possible that the application of bacteriological methods in the study of grahamellae may serve to simplify the situation and prove the identity of some of the organisms heretofore recorded as separate species.

III. Haemobartonellae of Small Mammals

Haemobartonella infections following splenectomy of the common vole, the local deer mouse, the gray-backed deer mouse, the short-tailed shrew and the gray squirrel have been studied. While uniform failure has attended all attempts at cultivation, morphological study has revealed features of interest, which suggest hitherto unsuspected relationships.

Haemobartonella microti[5] Tyzzer and Weinman, 1939, of the Common Vole

This species has been restudied with reference to its culturability and to the question of its distribution in grahamella-like pattern in the red cells during certain phases of infection. Two strains of this organism were available, one from the same locality as the one formerly studied, *i.e.* Martha's Vineyard, Massachusetts, the other from a locality near Boston. In two passages of the former strain, induced infections were studied in five splenectomized voles and attempts were made to cultivate the haemobartonella, in two at the height of the infection and in one three months after splenectomy. No growth was obtained either on leptospira medium or on blood agar slants. The second strain was transferred to splenectomized white mice, some of which were injected at the same time with grahamella from cultures. While the grahamella did not become established in these mice, the haemobartonella was carried through six successive passages. *H. microti* occurred in rather large numbers in the blood of infected mice at the height of infection, but only a mild anemia developed and there were no symptoms. In the first transfer the infection did not become apparent in the blood until the elapse of a fortnight, but in subsequent transfers it had become quite heavy by the end of five days, and in some animals lasted for more than four weeks. The morphology of the strain of *H. microti* passaged in mice shows rather less variation than was presented by the strain which served for the original description of *H. microti*. Nevertheless, this strain is quite similar in its staining qualities and in the morphology of the individual organisms to the original strain, while the rafts of organisms extending over the surface of the red cells and the "stringed bow" arrangement which are characteristic of *H. microti* are found on search. Many attempts have been made to cultivate this strain of *H. microti*, but no growths have been obtained.

In our previous study, two splenectomized white mice inoculated with the blood of a trapped vole presented not only typical bartonellae but also on one occasion red cells containing rods having a grahamella-like distribution. Re-examination of the stained films of these two mice leaves no doubt as to the presence of characteristic grahamellae in the red cells. The possibility of a natural grahamella infection in these two animals may be considered, but in the great number of blood examinations made upon the stock of mice used in the course of our investigation, this is the only occasion upon which grahamella has been observed. In the light of the present study, our previous conclusions relative to the ability of this haemobartonella to assume a characteristic grahamella-like distribution require revision. While we have no proof that this may never occur, it now seems probable that the inoculation of the two mice in question with vole blood resulted in a mixed infection of haemobartonella and grahamella, notwithstanding the failure in the present investigation to produce visible grahamella infection in the white mouse. That the vole grahamella is able to survive following its injection into the splenectomized white mouse, when not microscopically demonstrable in the

[5] Yakimoff (1928) applies the name *Bartonella arvicolae*, without indicating it to be a new species, to an organism or possibly organisms of the European vole, *Arvicola arvalis*, studied by Zuelzer (1927). About the only indication furnished by the latter author that she is dealing with any organism other than the grahamella in the infections which she describes is the statement that in some instances from 80 to 90 per cent of the vole's red blood corpuscles presented bartonella-like inclusions. Her description applies to organisms seen in the blood of non-splenectomized voles, no suggestion is offered as to the possibility of mixed infection, and the vole's blood is reported not to be infective for rats and mice but infective for dogs. Irrespective of the status of *Bartonella arvicolae* as a species, the identity of any organism alluded to in Zuelzer's paper and the haemobartonella described in the local American vole appears to be improbable. In the present study very closely related host species have been found to harbor biologically distinct haemobartonellae.

blood, is evident by its subsequent isolation in culture from such animals in a number of instances, as already considered in detail under *G. microti pennsylvanici*.

Since in our former study (Tyzzer and Weinman, 1939) a culture was obtained which produced infection of the red blood corpuscles on injection into splenectomized mice, the possibility of survival of *H. microti* in a culture in which the growth really represented that of the vole grahamella was considered. In order to test the survival properties of the former, generous samples of the heart's blood of a mouse showing a heavy infection of *H. microti* was distributed into three tubes of leptospira medium which were then kept at room temperature. The bulk of the implanted blood was withdrawn from the tubes after intervals of five, ten, and sixteen days respectively, and that from each tube was injected subcutaneously and intraperitoneally into a splenectomized mouse. No infection resulted in any animal, indicating that the survival properties of the vole haemobartonella at room temperature are notably low.

In contrast to the ease with which the grahamella of the vole is isolated in culture, all attempts to cultivate *H. microti* have failed notwithstanding the implantation of leptospira medium, in which growth was reported in a previous paper, and of many other media with heavily infected bloods. In view of these consistent failures and of the frequent occurrence in the vole's blood not only of grahamella but of other types of organisms that grow readily in leptospira medium, it now appears improbable that the organism grown in our earlier investigation was *H. microti*. The original purpose of the present study, *i.e.* to clarify certain confusing points in regard to the differentiation of haemobartonella and grahamella infection in the common field vole, has been accomplished. Not only is there a distinct difference between *G. microti pennsylvanici* and *H. microti* in regard to propensity for growth on artificial media, but the propagation of pure infections of these organisms and associated morphological studies have also aided in correcting our earlier misinterpretation of a mixed infection of the two organisms.

Haemobartonella peromysci, new species, of the Local Deer Mouse, *Peromyscus leucopus novaboracensis*

Splenectomy performed on twelve trapped deer mice of the local species was followed by haemobartonellosis in six animals. In one of these, infection was delayed for such a long interval after splenectomy that the contraction of the infection in some unknown manner after the operation seems at least possible. In the other animals haemobartonellae appeared in the blood from two to six days and had disappeared after periods of from eleven to thirty-three days after splenectomy. The six animals in which splenectomy was followed by no evidence of natural infection were obviously younger than the other four, although approaching maturity. In these, induced infections appeared in from five to ten days from the time of inoculation, and organisms were microscopically demonstrable in the blood of four for periods ranging from forty-nine to sixty-three days. In one the infection had disappeared before the fourteenth day after inoculation. Temporary diminution and recrudescence occurred in those which presented infections of long duration. In one animal there were three relapses noted. Nine deer mice of this species, born in captivity, developed no infection following splenectomy. Subsequently induced infections in four of these became apparent in from eight to eleven days after inoculation, and ran a similar course to the induced infections in splenectomized trapped animals. The haemobartonellae increased rapidly from the time of their first appearance in the blood, either following splenectomy of an infected animal or the inoculation of a splenectomized clean animal; the infection had become heavy in one instance two days after splenectomy, and in inoculated animals it usually reached its height in nine or ten days. This rapid increase usually terminated in a crisis characterized by the appearance of a severe anemia and the diminution, or even disappearance, of organisms from the blood. In some instances the initial crisis marked the termination of visible infection, but more frequently the organism subsequently became again apparent in numbers increasing until the precipitation of a second blood crisis, this cycle being repeated several times in long infections. Notwithstanding the severity of the anemia produced, the infected deer mice did not appear definitely ill or especially weak, and such deaths as occurred in the series were not attributed to bartonellosis.

The infection of the local deer mouse was transmitted to splenectomized animals of other species, *e.g.* white rats, white mice, and voles, by the injection of infected blood. Brief infections

were produced in two of four half-grown splenectomized white rats injected. The inoculation of two splenectomized voles resulted in severe infections, one dying twelve days after inoculation. The other became extremely weak, with reticulocytes constituting more than 40 per cent of the red cells at the end of a fortnight, but had recovered by the end of three and a half weeks after inoculation.

The haemobartonella of the local deer mouse was passaged from deer mouse to white mouse to deer mouse, and subsequently through seven transfers in white mice, all animals being previously splenectomized. It would appear that inoculation of white mice followed by splenectomy furnishes visible blood infection only if the operation is performed soon after the inoculation. Thus, a splenectomy nine days after inoculation was followed by a rather heavy haemobartonellosis, but a splenectomy performed at the end of sixteen days was followed by no demonstrable infection. The haemobartonellosis induced in splenectomized white mice is commonly severe and occasionally fatal. No attempt has been made to determine the effects of variation of dosage of infective material. In the course of routine inoculations in which blood with large numbers of haemobartonellae is injected, the organisms appear promptly in the blood, frequently within an interval of four days, although on occasion they have made their first appearance as late as thirteen days after inoculation. There is a rapid increase in the number of haemobartonellae, which terminates in a blood crisis with a suddenly developing anemia and diminution or disappearance of organisms. The mice at this time are somnolent, weak, and show extreme pallor of the exposed skin, with staring of the fur. Hemoglobinuria has not been observed and the urine remains pale in color. Fecal pellets are passed in long strings, due to excess of intestinal mucus. Gangrene of a portion of the tail frequently follows the snipping of the tip in obtaining blood for films. The blood is dull-colored and so thin that it is difficult to prepare satisfactory films. Hemoglobin, as estimated by Tallquist's scale, is commonly below 40 per cent normal. The body temperature becomes very low, rectal readings being frequently below 32.5° C. Notwithstanding the severity of the infection, the majority of the mice show improvement within a few days and, though they may pass through several successive crises, eventual recovery supervenes in most cases. Death occurs most frequently in the initial crisis, and in one instance by the ninth day after inoculation, but has occurred as the result of a severe crisis as late as ninety days after inoculation.

Morphology.—Haemobartonella peromysci, as compared with *H. microti*, is very delicate, especially as it appears in the blood of its natural host, so that without intense staining it here might very readily be overlooked. It presents certain characteristics which have apparently not been recognized or their significance not fully appreciated in the haemobartonellae of rodents. Interpretations of stages of development are based on the assumptions of growth and multiplication. The organism in its initial growth on the red cell presents filaments, a large proportion of which are branched (Fig. 15 *A*, *B*, and *C*), and which are commonly found radiating from a central granule or from a faintly stained area (Fig. 14 *A*, *B*, and *D*). Frequently a definite, although rudimentary, mycelium is formed. Later the mycelium or filaments are broken up into small elements that may become scattered over the surface of the red cell or concentrated as a colony on a limited area of its surface. The filaments in their early development appear practically homogeneous but later become definitely beaded and show coccoid or rod-like elements which arise within the filament rather than from a clearcut fission. The filaments may be straight or slightly or markedly tortuous, and frequently associated with them are sheets of protoplasmic growth which stain less intensely and which may present a faint reticulum or appear granular. These sheets may appear very early with rudimentary filaments radiating from the periphery, or they may be situated on well-developed filaments or even rods. When situated laterally on filaments, the development of the latter on the border of the sheet would be conceivable, but such sheets also occur frequently as expansions of portions of the filaments. It is difficult to escape from the conviction that there is a high degree of plasticity in the development of this organism. It would seem probable that such sheets may arise from small elements such as rods and coccoids as well as from filaments.

Transfer to foreign hosts serves to accentuate certain morphological features of *H. peromysci*. In the development of *H. peromysci* in its natural host, *Peromyscus leucopus*, ring forms are rare and in many preparations not demonstrable, and this is also true of its development in the white rat (Fig. 14 *C*) and the vole (Fig. 14 *B*). In the

common albino mouse, however, ring forms occur in such numbers and so well developed as to suggest a mixed infection (Fig. 14 *D* and Fig. 15 *D*). They may form within filaments or from smaller elements which arise from the latter. With the development of the organism on the red cell, the

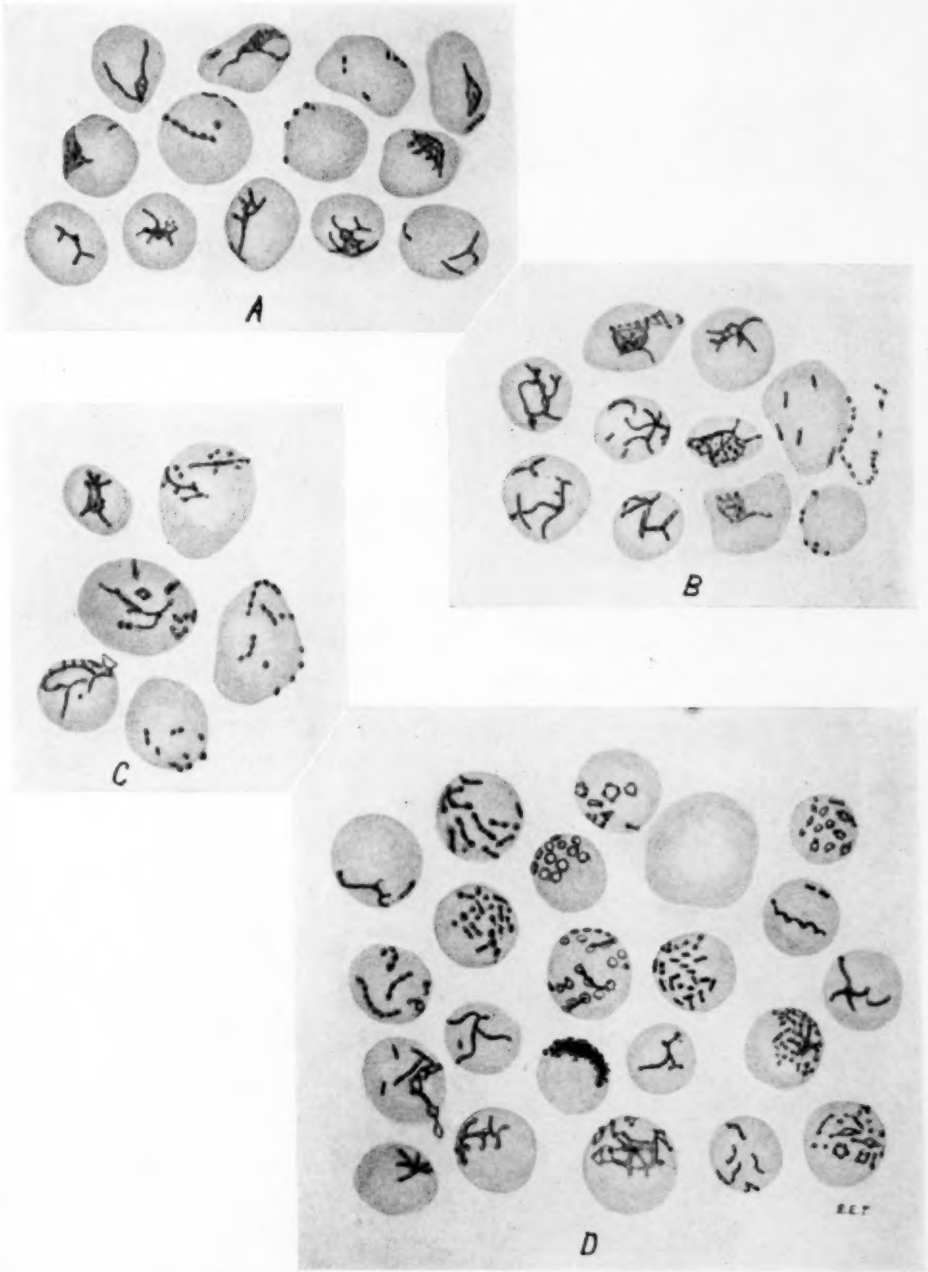


FIG. 14. *Haemobartonella peromysci* of the local deer mouse—selected erythrocytes drawn at 2000 diameters. *A*. From the blood of the natural host, showing branching forms originating in deeply stained granules, also chains, sheet-like growths and isolated elements. *B*. The same organism on transfer to the vole. *C*. Infected erythrocytes from an experimentally infected white rat. *D*. Morphological modification with excess of coccoids and ring forms on transference to white mouse.

filaments may give rise to a large number of coccoids and rods either of which may become expanded and show indistinct outlines. While the ring forms apparently may arise from any of these elements, the larger rings apparently develop from the indistinct or "blurred" forms, often appearing uneven with several granules or concentrations of the protoplasm. The significance of the ring forms which constitute such a prominent feature of the infection in the white mouse is not clear, but there is at present no basis for assuming that they represent degeneration

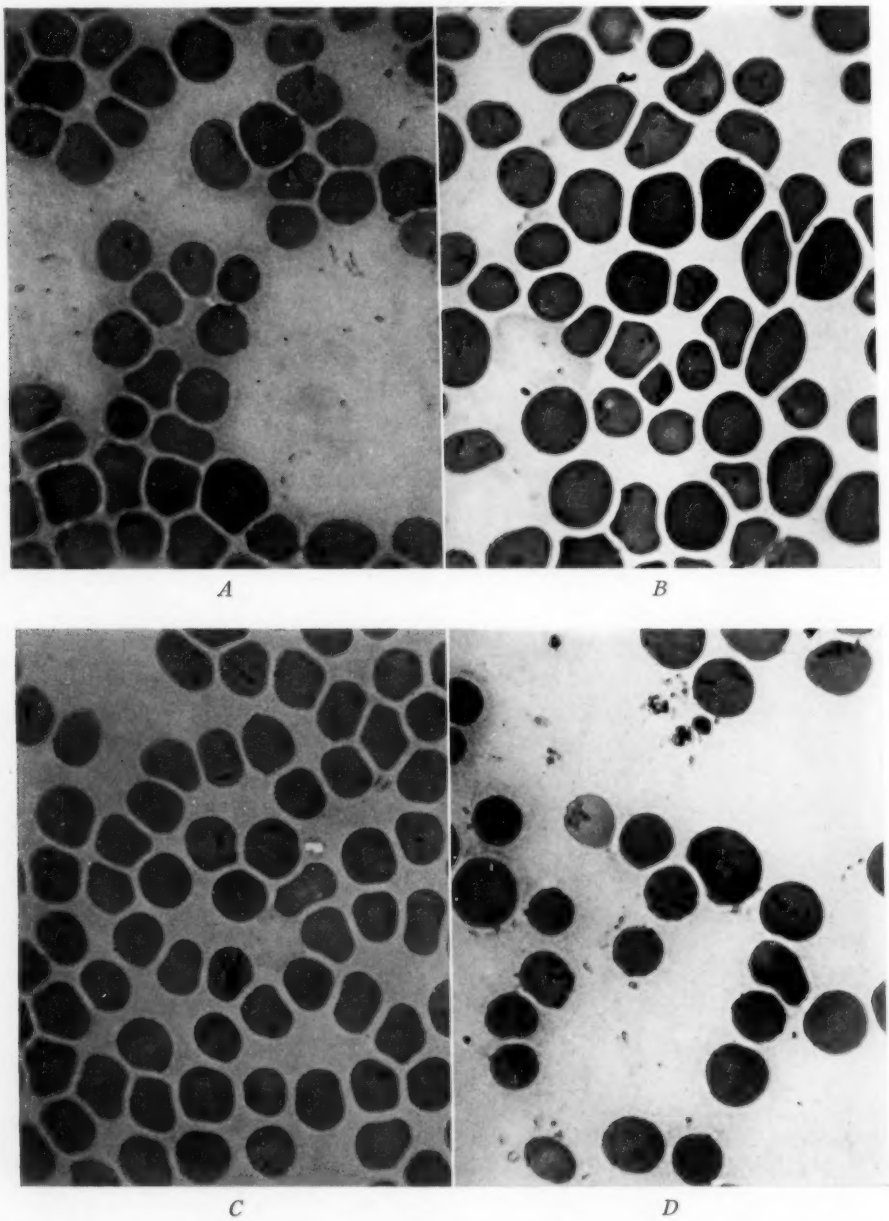


FIG. 15. Giemsa-stained blood films from animals infected with *Haemobartonella peromysci*, taken at 1275 diameters, showing branching forms in *A*, *B* and *C*. *A*. Natural infection in local deer mouse, 7 days after splenectomy. *B*. Induced infection in splenectomized vole. *C*. Induced infection in splenectomized local deer mouse. *D*. Induced infection splenectomized white mouse—coccoid forms in blood showing severe anemia.

forms. Rods or filaments from which rings are forming become more and more indistinct. Coccoids, rods and rings are rather evenly distributed at intervals, generally over a portion of the surface, less frequently over the entire surface of red cells, but occasionally are accumulated in a dense mass at the margin of the cell. A single cell may show long and short rods and all stages between minute coccoids and coarse rings. The infection in the white mouse also presents markedly tortuous filaments, shorter elements which are sharply bent and also expanded growths with processes anastomosing to form a coarse web. Induced *H. peromysci* infections of the vole present rudimentary mycelia, branching filaments and rods, but few coccoid forms, and ring development is practically negligible. Deeply stained microcytes are often largely covered with organisms. In this animal the number of organisms is maintained at a high level after the development of anemia. The blood of the infected rat presents filaments, many of which branch, while the smaller elements are predominantly coccoid in morphology. Deeply stained microcytes are often heavily infected and may then present an irregular outline.

At the time of the blood crisis, whether of the deer mouse, common white mouse, or vole, there is a change in the shape of the infected red cells which tend to become globular. The lack of uniformity in the diameter of the red cells as seen in dried films is not, however, based wholly upon change in shape, for the globular erythrocytes seen in fluid blood at this time vary considerably in diameter. In stained blood films considerable numbers of free organisms occur. These have a less distinct outline than those attached to the red cells and in general appear larger, some being obviously distorted. Early in the anemic period when reticulocytes are abundant, the haemobartonellae are not found to any extent on this type of cell but are distributed quite generally on the red-staining erythrocytes. With the development of an extreme anisocytosis, both the large reticulocytes and the intensely staining microcytes are seen with great numbers of attached organisms.

Haemobartonella peromysci maniculati, new variety, of the Gray-backed Deer Mouse, *Peromyscus maniculatus gracilis*

A haemobartonella appeared after splenectomy in three gray-backed deer mice, two of these animals showing a mixed infection with an eperythrozoön, and the third a triple infection with an eperythrozoön and a grahamella present. The time of appearance of the haemobartonellosis showed wide variance, five days after splenectomy in the animal with the triple infection, and forty-one and forty-six days respectively in the other two. It seems probable that the long delay in the development of this infection in the latter two animals is attributable to the incidence of an associated eperythrozoön infection. The effect

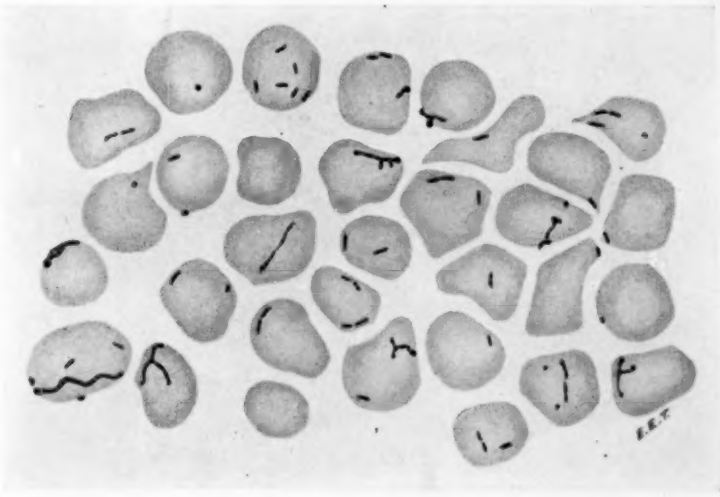


FIG. 16. *Haemobartonella peromysci* (var.) *maniculati* in blood of splenectomized gray-backed mouse. Note filamentous and branching forms. Drawn at 2000 diameters.

of one infection upon another will receive further consideration later on.

The haemobartonellosis, after making its appearance in the gray-backed mice, was followed in two of these animals over a period of five weeks during which severe anemia was noted on various occasions. The infection was passed to a common deer mouse in which it persisted for more than sixty days after inoculation. A small amount of this animal's blood showing great numbers of organisms was injected into two splenectomized white mice without resulting infection. Another splenectomized white mouse which was injected with blood from a gray-backed mouse at the height of its haemobartonellosis also failed to develop visible infection. Hence it appears that the haemobartonella of the gray-backed mouse differs from that of the common deer mouse in that it is relatively non-infective for the splenectomized white mouse.

Notwithstanding the very pronounced biological differences already noted, the morphology of this haemobartonella is so similar to that of the common deer mouse that no distinctive feature has been found for their microscopic differentiation (compare Figs. 14 *A* and 15 *A* with Figs. 16 and 17), although the former organism as it occurs in its natural host appears to be in general more slender. Filamentous and branched forms occur in considerable numbers, the coarser filaments appearing to arise from a rounded granule, and short filaments radiating from a more faintly stained central sheet are occasionally seen. Delicate rods constitute the preponderating form although minute coccoids occur in small numbers. Many of the rods are even more slender than any of the grahamellae of the hosts studied. This organism when transferred to the common deer mouse presents a remarkable morphological transformation. In this species it is comparatively coarse and, although filaments are present, large coccoids occur in great number, sometimes in chains but frequently spaced at regular intervals on the surface of the red blood corpuscle (Fig. 18). A few ring forms such as occur in *H. peromysci* when transferred to the white mouse have been found.

Haemobartonella blarinae, new species, of the Short-tailed Shrew, *Blarina brevicauda*

A trapped shrew was splenectomized at the end of five and a half weeks and survived for a total of two and a half months after capture. Grahamella-infected erythrocytes were found previous to splenectomy. Four days after operation, blood films showed a haemobartonellosis of moderate intensity with some indications of anemia, *e.g.* marked anisocytosis and large num-

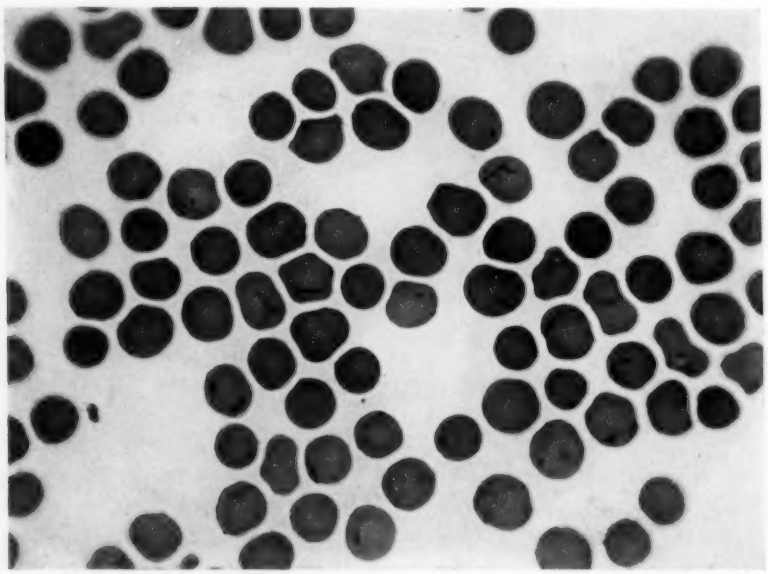


FIG. 17. Branching forms of *H. peromysci maniculati* in blood of gray-backed mouse 41 days after splenectomy. This infection replaced an erythrozoön infection which followed splenectomy. × 1275.

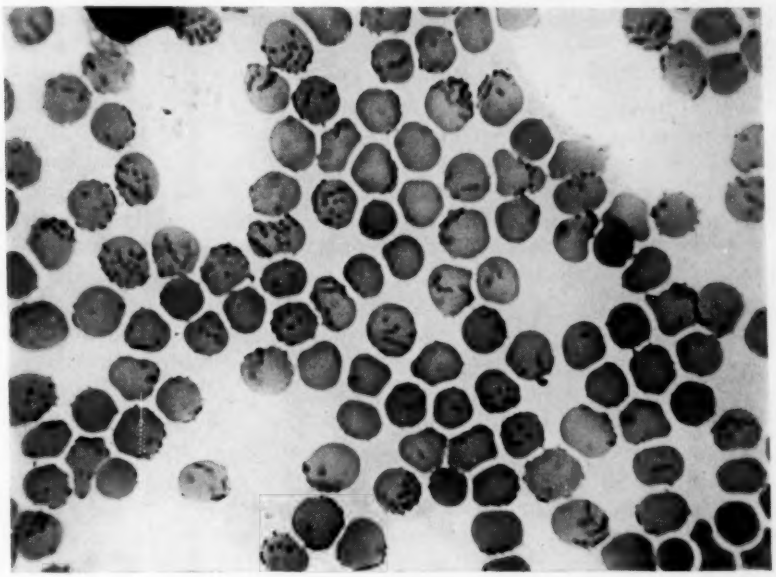


FIG. 18. The same organism as in Fig. 17, 9 days following its transfer to a splenectomized *Peromyscus leucopus novaboracensis*. Note morphological transformation. × 1275.

bers of reticulocytes. Blood planted on various media before as well as after splenectomy furnished a growth which, though not tested by inoculation, probably represents the grahamella. The inoculation of two splenectomized local deer mice with blood taken from this shrew before its splenectomy and of two splenectomized white mice with blood taken at the height of its haemobartonellosis failed to produce infection of any sort.

Another shrew splenectomized four weeks after being taken in a trap also survived for a period of two and a half months. The blood presented haemobartonellae three days after splenectomy, the infection soon became intense, and, although showing some degree of fluctuation, was maintained at a relatively high level until death. No grahamella-infected red cells were found either before or after splenectomy. While there was quite a severe anemia in this animal throughout the course of the infection, the only symptom noted was temporary loss of appetite on two occasions. Leptospira medium and blood agar slants planted with splenic tissue at the time of operation and a great variety of media planted with blood twelve days later furnished no growth.

The haemobartonella which appeared in the blood of the above two shrews in three and four days after splenectomy shows in the earlier stages of infection extreme polymorphism, with delicate rods and coccus-like forms, often occurring in chains but with a large proportion of coarser elements, each with a deeply stained bead-like granule. The blood picture ten days after splenectomy shows a marked change. The infection is now extremely heavy so that the greater proportion of the red cells show thickly distributed organisms, with relatively few of the coarse forms seen earlier. Microcytes are encountered, some completely covered with organisms as to be scarcely recognizable.

On studying the coarse forms conspicuous in the early stages of infection, it is found that they appear as thick bands or filaments stretching across the red cells and quite regularly show a bead or globule, situated either at the end or at some point along the course of the filament (Figs. 19 and 20 *A*). These thicker filaments usually take a somewhat more bluish tint with the Giemsa stain than the more delicate forms which are a dull slaty violet when not stained intensely. There are, however, all grades intermediate between the coarser and the more delicate organisms. The terminal bead in the coarse forms, when not too intensely stained by the Giemsa method, is distinctly reddish in contrast with the bluer substance of the filament. On further inspection, very short filaments are found evidently

sprouting from reddish globular bodies (Fig. 19, above and to left) which thus seem to be of the nature of spores comparable with certain of those of the higher fungi. The filaments frequently show other granules, there usually being one at the growing tip, but such granules are readily distinguished on the basis of their coloration, at least in the coarser filaments, from the reddish germinative globule. The coarse filaments frequently appear as straight bands extending across the red cell, but may attain such length as to project beyond the margin of the latter or even across an adjacent cell (Fig. 19). Such extension of long forms beyond the cells to which they are attached possibly represents a partial detachment resulting from the dragging of the cells in the preparation of the blood film. The coarse forms are frequently clearly banded or they may appear as a string of coarse beads. Branching occurs, the tips of the branches often being expanded.

The more delicate forms of the organism are best studied in the fully developed infection which presents a bewildering variety of types and distributions. Rods and filaments predominate over rounded forms but, such is the intensity of the infection, all types occur in great numbers. Frequently the organisms are spaced at regular intervals on the surface of the red cell, but often there is a localized growth resulting in a dense colony appearing as an intensely stained cap on some portion of the cell's surface. Rudimentary mycelia showing branches radiating from a central portion containing a reddish-stained granule are well represented. Filaments, rods, coccoids and granular sheets show a wide range in size and in the manner of their distribution. Filaments and rods are sometimes arranged along parallel lines as though wound about the red cell but are also scattered indiscriminately. Thick smooth rings may develop from the coccoid form, but most ring forms present an uneven contour due to the presence of granular thickenings. Frequently such forms appear quadrangular, triangular or bipolar. In several instances rings were also observed distributed along a filament.

In addition to and among the above forms, more expanded, less intensely stained material is found which may be globular or have an uneven and in some instances an ill-defined contour. This reddish material may occur at the ends of mycelial branches or as expansions in a filament and probably also through the transformation of rods or rounded forms. Its similarity to the spore-like bodies in regard to staining qualities as well as the indication that it is a product of growth and development suggests the possibility that it presents the beginning and end of a growth cycle. Thus an anastomosing mycelium may extend from an aggregate of such reddish material (Fig. 19 to right), or filaments may radiate from a central reddish mass. While a large

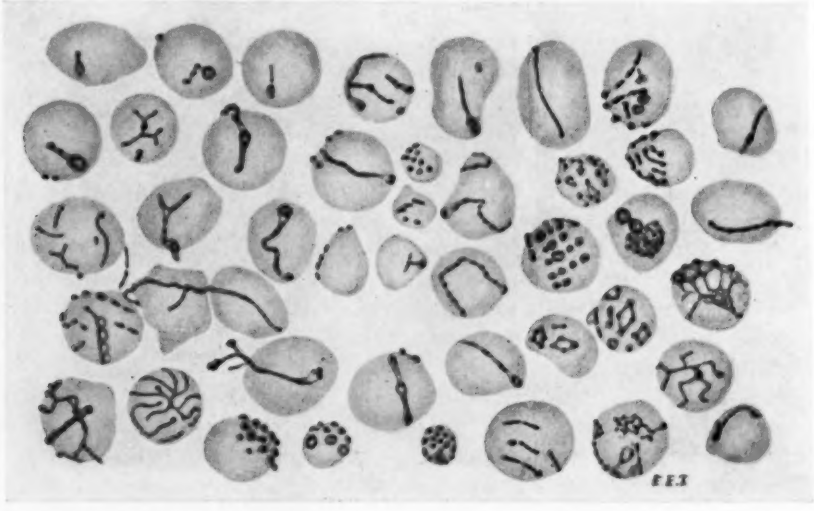


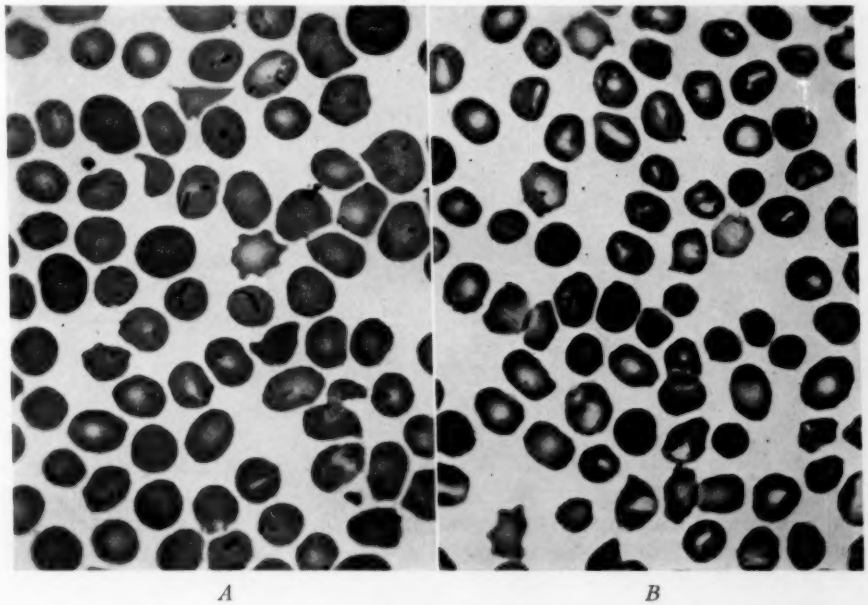
FIG. 19. *Haemobartonella blarinae* of the short-tailed shrew. Selected erythrocytes with various forms of organism. Note coarse filaments, often banded, springing from spore-like bodies, branching and extending beyond the margins of the erythrocyte—drawn at 2000 diameters.

proportion of the simple coarse filaments arise from a well-defined spore-like globule, others arise from an ill-defined reddish mass. This organism, with the luxuriance of its growth in the blood of the shrew and the relatively large size of some of its developmental forms, has furnished additional features indicating the fungus-like character of the haemobartonellae studied.

Haemobartonella sciuri, new species, of the Gray Squirrel, *Sciurus carolinensis leucotis*

A rather light bartonellosis developed in a trapped gray squirrel in ten days after its splenectomy. Four days later the infection had disappeared, was again in evidence after an interval of eight days, but never again became apparent during subsequent observations. The injection of three normal white mice with a suspension of ground splenic tissue produced no demonstrable infection. No attempt was made to transfer the infection to splenectomized mice.

This haemobartonella presents a wide range of variation and assumes rather remarkable forms. Minute rods are of common occurrence and related to these are filaments of various lengths, some continuous, others segmented, often stretching across the face of the red cell and bent over its margin. The rods and filaments vary in thickness, many in the course of their length being of uneven thickness, and some become very coarse. Within such thickened forms, rounded, often oval bead-like elements develop (Fig. 21). Thus, a filament may be transformed into a beaded chain. The bead-like elements stain a dull reddish at the periphery, but are otherwise faintly stained in contrast to the intensely staining basophilic rods and filaments. Some of the rounded forms have the appearance of large thick rings. Similar beads and rings sometimes arise within rather slender, deeply staining rods, and simulate very closely spores developing within bacilli, but also occur in greatly thickened rods which often stain rather faintly and have a reddish tint. The oval spore-like bodies, with the rod extending lengthwise over its surface, constitute a striking characteristic of the infection. The evidence of the origin of the bead-like and coarse ring forms from rods and filaments is quite conclusive, but their significance is not understood. While they resemble the spore-like forms of the shrew in their staining qualities, no sprouting of filaments from them has been observed. Three beaded filaments radiating from a central granule are shown in Fig. 21, above and to right, and in other instances two divergent filaments are found having a common origin in a single granule.



A *B*

FIG. 20. *A*. Band forms, *H. blarinae*, in film taken at an early stage of infection in the splenectomized shrew. × 1275. *B*. Branching forms, later in same infection. × 1275.

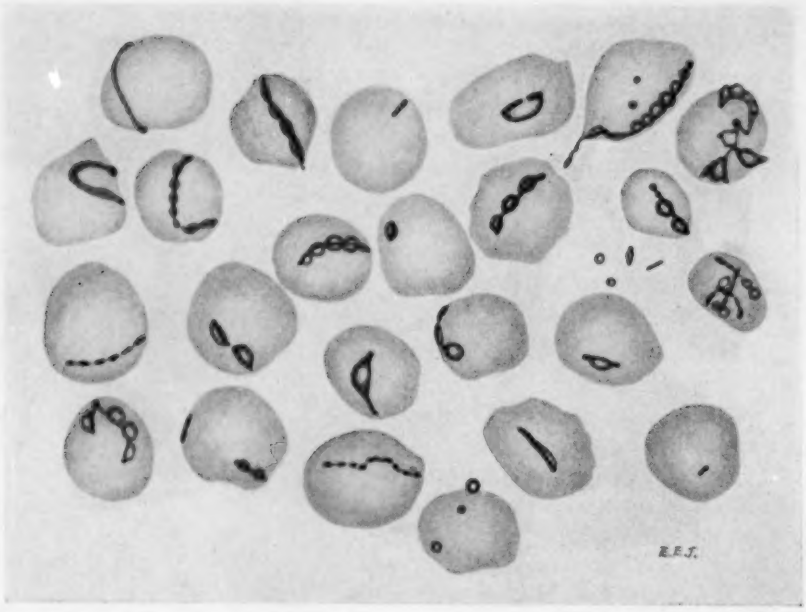


FIG. 21. *Haemobartonella sciuri* of the gray squirrel. Coarse pale-stained beads formed on rods and filaments. Above and to right three filaments radiating from a single granule. × 2000.

Nauck (1927) observed an infection in twelve of a series of twenty *Sciurus vulgaris* splenectomized in Pekin. The organism which he thought might be a bartonella occurred in great numbers in the blood, as rings, rods and comma-shaped forms of variable size, nearly all the red cells being infected. Inoculation of uninfected splenectomized squirrels was successful but the white rat proved refractory.

Indications as to the Systematic Position of Haemobartonellae

Certain well-known characteristics of haemobartonellae, such as their prevalence in a latent condition in various species of animals, their rapid increase following splenectomy attended with the development of severe and often fatal anemia, and the eradication of infection by arsphenamine therapy, have already been discussed. This type of organism differs from *Bartonella bacilliformis* of the human being in that no multiplication in any tissue apart from the blood is known to occur, there are no eruptions associated with infection, and survival outside the body is of brief duration. The present study has served to emphasize the difficulty of cultivation, no success having attended numerous attempts utilizing media upon which *B. bacilliformis* and grahamellae are readily grown.

The chief contribution, however, has been derived from comparative morphological studies of the haemobartonellae of various animals. The haemobartonellae of the two species of deer mice studied and that of the short-tailed shrew all show definite branching in their growth on the surface of the red blood corpuscles, and also evidence of sprouting from a spore-like granule. There is a suggestion of septa in the coarser filaments but multiplication is accomplished largely by the formation of bead-like elements within the substance of the filament. The haemobartonella of the common gray squirrel presents huge, pale-red, spore-like bodies originating in rods and filaments, and may show several filaments radiating from a single granule, but no branching has been observed. Doubtless a more extended study of this organism would reveal additional features. No branching has been observed in *H. microti* although thick fungus-like filaments occur in both vole and mouse blood. In the vole there are short thick forms with a terminal globule suggestive of sprouting from a spore.

For comparative purposes preparations of *H. canis* and *H. muris* also have been studied. In the single film available, the former organism

presents moderate numbers of branching filaments and numerous reddish globules and rings. *H. muris* is represented chiefly by minute elements with no notable tendency to form filaments and hence shows no branching. There are occasional coarse forms with a terminal bead. This organism, the type species of the genus, while presenting the other characteristics of the group, lacks those morphological peculiarities which in certain haemobartonellae furnish a clue as to their systematic position. From the descriptions here furnished it will be obvious that various species found in small mammals present features characteristic of higher fungi, such as the development of branched filaments from spore-like granules and even the formation of a rudimentary mycelium. Doubtless the successful cultivation of haemobartonellae would yield additional information as to their nature. Notwithstanding the indications of relationship with the higher fungi, it is probably well to defer the further classification of the haemobartonellae until the possibilities of cultivation are given further consideration. Thus, from a comparison of the haemobartonellae of small rodents and *Bartonella bacilliformis*, evidence is obtained of a phaenotypic rather than a phylogenetic relationship.

IV. Eperythrozoa of Small Mammals

Eperythrozoa of the Common Mouse and of the Local Vole

Eperythrozoön coccoides Schilling, 1928, of the common mouse occurs with great frequency in healthy laboratory stock mice, its presence being made manifest by splenectomy. The peculiar property of this organism in preventing or modifying certain haemobartonella infections will be discussed later. This organism is only slightly infective for splenectomized voles, one very light and one moderate but transient infection following the injection of four voles with heavily infected mouse blood. An eperythrozoön of the local vole, probably of the species *E. dispar* described in the European vole, *Arvicola arvalis*, by Bruynoghe and Vassiliadis, 1929, on the other hand, appears from past experiments not to be transferable to splenectomized white mice (Tyzzer and Weinman, 1939). This species as compared with that of the common laboratory mouse presents a relatively large proportion of coccoid forms and few rings. No natural eperythrozoön infection has been observed in fourteen of the local species of deer mice splenectomized, but three gray-backed deer mice from Ossipee, New Hampshire, all showed eperythrozoa after splenectomy.

Eperythrozoön varians, new species, of the Gray-backed Deer Mouse, *Peromyscus maniculatus gracilis*

This species shows a greater variety of forms than either *E. coccoides* or *E. dispar*. At the height of the infection organisms occur in great numbers in the plasma, relatively few being attached to the erythrocytes (Fig. 23). At this time, there are rings, coccoids of various sizes, some of the latter being extremely minute. It appears probable that the minute coccoids may develop into rings and also that the elongate, diplococcoid forms and tetrads may have to do with multiplication (Fig. 22). Slender, faintly staining bacillary forms also occur in the plasma and are also seen attached by one end to the margins of the red cells from which they extend usually nearly vertically (Fig. 24). Wherever an organism comes in contact with the surface of a red cell, it stains intensely. Thus, rods having one end in contact with a red cell present an intensely stained terminal granule, while organisms lying flat against the margin of the red cell are stained intensely throughout, or on one side.

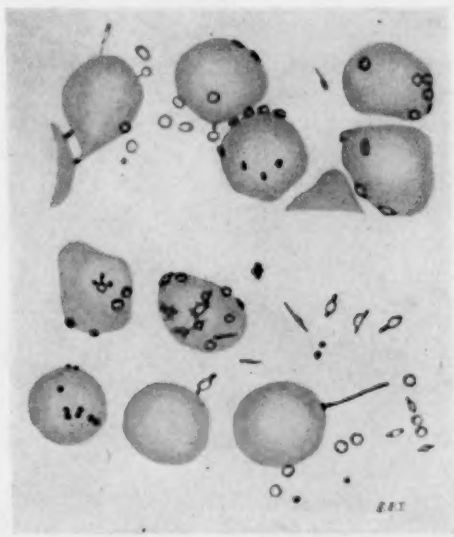


Fig. 22. *Eperythrozoön varians* of gray-backed mouse. Above an area of blood film showing intense staining of organisms where they are in contact with erythrocytes. Below selected cells and organisms showing variability in size and outline. Small coccoid forms in pairs or tetrads are probably multiplication forms. × 2000.

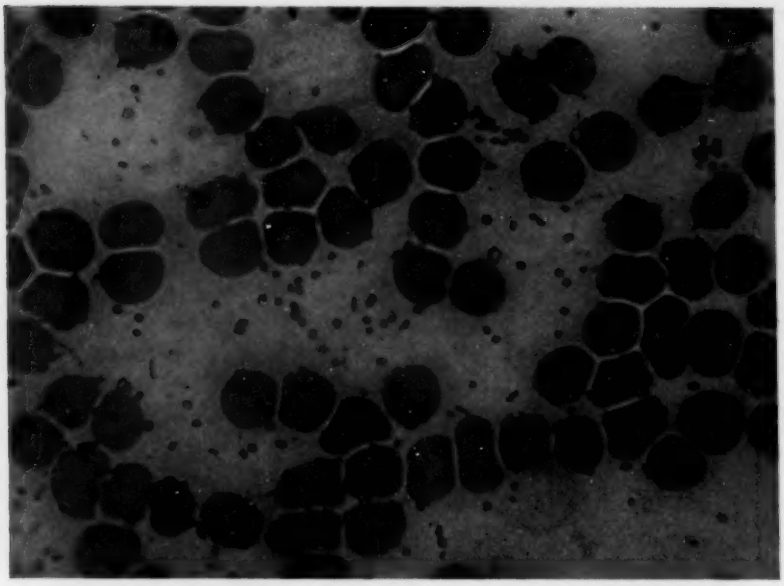


FIG. 23. *Eperythrozoön varians* of gray-backed mouse in blood of splenectomized local deer mouse, 9 days after inoculation. Note elongate forms as well as rings. × 1275.

Many of the bacilliform elements show an unstained lens-like swelling, indicating the formation of a ring within the substance of the rod. Pulled-out, elongate forms of uneven thickness and occasional organisms having an angular process at the side are seen. Pairs and chains of lens-like, coccoid and pulled-out forms occur, which may be explained on the basis of adventitious grouping and distortion. The occurrence at the end of the film of small, amorphous elements corresponding in distribution and size to the well-defined eperythrozoa which they replace is obviously due to some artifact in preparation. It would appear that the physical properties of this organism are such that distortion forms are much more readily produced than is the case in the haemobartonellae studied.

This eperythrozoön appeared following splenectomy in three gray-backed deer mice, in one in mixed infection with haemobartonella and grahamella, in the other two in association with latent haemobartonella alone. In the latter two animals, the eperythrozoön had not appeared eleven days after splenectomy, but was present in heavy infections by the twentieth day, which thereafter gradually diminished in intensity and were replaced by the end of forty-one and forty-six days respectively by haemobartonella infections. The eperythrozoön infections were associated with a notable degree of anemia, as indicated by an increase in reticulocytes, but not approaching in severity that developing in the haemobartonellosis which followed. Attempts made at the height of the infection to cultivate the eperythrozoön on various media failed. A small amount of blood from the animal showing triple infection was injected into a splenectomized white mouse without result, but the inoculation of a splenectomized local deer mouse was followed by a haemobartonella infection soon replaced by an eperythrozoön infection. While both organisms were present in the blood of this animal, an attempt was made to passage the infections to two splenectomized white mice and from one of these to a third splenectomized mouse, but in none of the three animals did either type of organism make its appearance. Blood taken at the height of the eperythrozoön infection in one of the gray-backed deer mice was injected into a splenectomized local deer mouse, producing an extremely heavy eperythrozoön infection not followed by haemobartonellosis (Figs. 23 and 24). Thus, it is clear that this eperythrozoön is readily transferred to another species of deer mouse. Failure even in the few attempts to pass it to splenectomized white mice furnishes rather conclusive evidence that the eperythro-

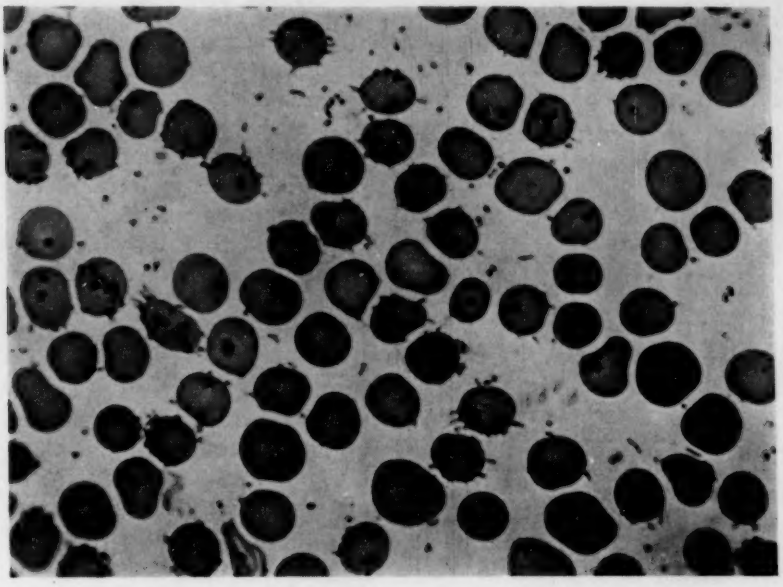


FIG. 24. Blood of same animal 21 days after inoculation, showing small coccoids and elongate forms extending vertically from the surface of the erythrocyte. × 1275.

zoön of the gray-backed deer mouse is distinct from *E. coccoides*.

V. General Considerations

Purity of Infections and Cultures

The purity of given infections is ever open to question, especially in case of those of the types under discussion which are quite frequently associated with one another in the same animal. The occurrence of mixed infections of haemobartonella and eperythrozoön is so frequent in nature that the possibility also of mixed infections of different species of haemobartonella may not be wholly disregarded. It would be ideal to produce infections by the injection of single organisms, but even if it were possible to accomplish this, the technical difficulties in isolating single organisms either within or attached to erythrocytes, appear to be very great. Isolation in culture should go far in the identification of various infections of the types studied. While the grahamellae of various species of animal have been found to grow readily on certain media, their cultural characteristics should be made the subject of further study. The readiness with which organisms of this type develop in leptospira medium planted with infected blood sets them off quite sharply from the haemobartonellae and the eperythrozoa. In the present preliminary study, rather definite differences have been noted in the gross cultural characteristics of grahamellae derived from different hosts. The morphological variation of the cultivated organisms is so extreme as to raise the question as to purity of the culture strains. The variation in the type of colony associated with slight differences in the media or in the inoculum is also a matter of interest. Culture strains derived from various types of colonies picked from blood agar slants appear in all respects identical and the conclusion is reached that we are dealing with pure rather than mixed cultures. Pure line cultures derived from single organisms would, however, present certain advantages. Less exact methods for eliminating certain organisms from mixed infections, such as repeated passage in a foreign host for the elimination of grahamella or arsphenamine treatment of the infected animal for the elimination of haemobartonella and eperythrozoön, are useful if properly checked.

The possibility of the occurrence of organisms of the "L" or so-called "pleuropneumonia type" in the animals employed or even in the cultures derived from them is by no means eliminated. While the laboratory-reared stock utilized has never shown any indication of disease, swellings of the feet and inflammation around the molar teeth have been observed in trapped voles kept

over long periods. Since the status of the "L" type organisms appears not to be definitely established, and since their study requires an entirely different approach with the employment of special media and techniques, it has not as yet been undertaken.

Host Specificity of Grahamellae, Haemobartonellae and Eperythrozoa

The results obtained in attempts to transfer grahamella infections to host species other than the ones in which they have been found demonstrates that this may be accomplished in closely related animals such as the two species of deer mice employed (Table 3, Nos. 9797 and 9800), and also occasionally with species that seem quite distant from one another, as the vole and the common white mouse. In two instances the inoculation of vole's blood showing both haemobartonella and grahamella into splenectomized white mice has produced mixed infections with grahamella, in which, however, grahamellae were apparent only on one occasion. *Grahamella pennsylvanici* may also survive in animals in which it is not microscopically demonstrable. Thus, in recent experiments, it has been recovered in culture after a sojourn of five weeks in splenectomized mice, in none of which were infected red cells found at any time.

Data relating to attempts to infect various species of animals other than the natural host are presented in Table 2.

The series of animals inoculated with the grahamellae of other host species are obviously too small to furnish more than suggestive results when certain factors involved in the experimental procedure are taken into account. The intracorpuscular situation of the organisms, as well as their being introduced into the test animals within corpuscles of a foreign species, may have a bearing on the success of the inoculation. It would seem that the probability of organisms contained within cells of a foreign host being destroyed by phagocytosis should be greater than would be the case with similar organisms free of such cells. Furthermore, the injection of blood containing infected red cells is not invariably successful for the transmission of infection to susceptible animals of the same species. The injection of grahamellae from newly isolated cultures would probably constitute a more reliable test and too few animals have been injected with cultured grahamellae from other species to establish host specificity as a wholly reliable basis for differentiating species of grahamellae. The distinctive characters observed in grahamella of various host species when propagated in culture media offer more conclusive evidence as to the multiplicity of grahamella species.

Certain of the haemobartonellae are readily transferred to splenectomized animals of species other than those in which they occur naturally. Some species are uniformly susceptible, *e.g.* the white mouse to *H. microti* of the vole and to *H. peromysci* of the local deer mouse. The splenectomized white rat appears non-susceptible to the former and slightly susceptible to the latter organism, presenting transient infection in a certain proportion of young splenectomized animals. Haemobartonellae that are morphologically indistinguishable, such as those of the two species of deer mice studied, show definite biological differences, one being non-infective for splenectomized white mice, the other being uniformly infective and pathogenic. Profound modifications in morphology have attended their transference to certain foreign hosts. Such changes have been observed in *H. microti* and *H. peromysci* when transferred to the common albino mouse and in *H. peromysci* var. *maniculati* transferred to *Peromyscus leucopus novaboracensis*.

In case of eperythrozoön infections, *E. coccoides* of the common mouse may produce transient infection in splenectomized voles, as shown in recent experiments, but the *E. dispar* of the vole has not infected splenectomized white mice (Tyzzer and Weinman, 1939). The eperythrozoön, *E. varians*, of the gray-backed deer mouse is readily transferred to the local deer mouse but not to the splenectomized white mouse. It would appear that infectivity for various host species should prove to be of value in the differentiation of species or varieties of the latter two types of organism under consideration.

Interference Phenomenon in Associated Infections

During the study of the infections above described, it was noted that the haemobartonellosis induced in the splenectomized white mouse by inoculation with *H. peromysci* of the local deer mouse, was made latent for long periods by preexistent eperythrozoön infection in the inoculated animal. Furthermore, if white mice which were already infected with the haemobartonella in question were inoculated with *E. coccoides*, the former organism promptly disappeared from

TABLE 2

SUSCEPTIBILITY, AFTER SPLENECTOMY, OF OTHER THAN NATURAL HOST SPECIES TO VARIOUS GRAHAMELLAE, HAEMOBARTONELLAE AND EPERYTHROZOA*

Type of Organism	Inoculum**	Host of Origin†	Species Inoculated	Number		Remarks
				Inoculated	Infected	
Grahamella	Blood	*M. pennsylvanicus pennsylvanicus*	White Swiss mouse	5	2	
Grahamella	Blood	*M. pennsylvanicus pennsylvanicus*	White Swiss mouse	7	0	Grahamella had disappeared from donor's blood.
Grahamella	Blood	*M. pennsylvanicus pennsylvanicus*	White rat (Wistar)	2	0	One animal half-grown; the other mature.
Grahamella	Culture	*M. pennsylvanicus pennsylvanicus*	White Swiss mouse	8	0	Grahamella isolated in culture from blood of one.
Grahamella	Culture	*M. pennsylvanicus pennsylvanicus*	Ibid., nursing	8	0	These not splenectomized.
Grahamella	Culture	*M. pennsylvanicus pennsylvanicus*	*P. leucopus novaboracensis*	1	0	
Grahamella	Blood	*P. leucopus novaboracensis*	White Swiss mouse	10	0	
Grahamella	Blood	*P. leucopus novaboracensis*	White Swiss mouse	8	0	Grahamella latent or very rare in donor.
Grahamella	Blood	*P. leucopus novaboracensis*	*Phyllotis darwini linatus*	2	1?	Recipients not laboratory-reared.
Grahamella	Culture	*P. leucopus novaboracensis*	White Swiss mouse	1	0	
Grahamella	Culture	*P. leucopus novaboracensis*	*Macaca mulatta*	1	0	
Grahamella	Blood	*P. maniculatus gracilis*	White Swiss mouse	1	0	
Grahamella	Blood	*P. maniculatus gracilis*	*P. leucopus novaboracensis*	1	0	
Grahamella	Culture	*P. maniculatus gracilis*	*P. leucopus novaboracensis*	3	3	
Grahamella	Blood	*Blarina brevicauda*	White Swiss mouse	2	0	
Grahamella	Blood	*Blarina brevicauda*	*P. leucopus novaboracensis*	2	0	
Grahamella	Blood	*Pitymys pinetorum*	White Swiss mouse	1	0	
Grahamella	Blood	*Phyllotis darwini linatus*	White Swiss mouse	5	0	
Haemobartonella	Blood	*M. pennsylvanicus pennsylvanicus*	White Swiss mouse	27	23‡	Failure to infect one entire group of 3 indicative of non-infective inoculum.
Haemobartonella	Blood	*M. pennsylvanicus pennsylvanicus*	*P. leucopus novaboracensis*	8	0	One not splenectomized.
Haemobartonella	Blood	*M. pennsylvanicus pennsylvanicus*	*P. eremicus*	1	0	
Haemobartonella	Blood	*M. pennsylvanicus pennsylvanicus*	White rat	7	0	
Haemobartonella	Blood	*M. pennsylvanicus pennsylvanicus*	Dog	2	0	
Haemobartonella	Blood	*P. leucopus novaboracensis*	White Swiss mouse	40	35‡	The blood of one of 5 animals not showing infection was examined only once, one had received arsphenamine before inoculation, and 3 were injected with non-infective blood.
Haemobartonella	Blood	*P. leucopus novaboracensis*	White rat (Wistar)	4	2	Light transient infections.
Haemobartonella	Blood	*P. leucopus novaboracensis*	*M. pennsylvanicus pennsylvanicus*	2	2	Severe infections, one fatal.
Haemobartonella	Blood	*P. maniculatus gracilis*	White Swiss mouse	3	0	
Haemobartonella	Blood	*P. maniculatus gracilis*	*P. leucopus novaboracensis*	2	1	One injected with blood showing E with H latent failed to develop bartonellosis.
Haemobartonella	Blood	*Blarina brevicauda*	White Swiss mouse	2	0	
Haemobartonella	Blood	*Blarina brevicauda*	*P. leucopus novaboracensis*	2	0	
Eperythrozoön	Blood	*M. pennsylvanicus pennsylvanicus*	White Swiss mouse	5	0	
Eperythrozoön	Blood	White Swiss mouse	*M. pennsylvanicus pennsylvanicus*	4	2	One light, one heavy, transient infection.
Eperythrozoön	Blood	*P. maniculatus gracilis*	White Swiss mouse	3	0	
Eperythrozoön	Blood	*P. maniculatus gracilis*	*P. leucopus novaboracensis*	2	2	Both infections intense.

* Animals tested for susceptibility in former investigation (Tyzzer and Weinman) included.

** The organism in question, demonstrated microscopically in the blood employed, unless otherwise stated, in most cases alone but in others with one or both the other types of organism apparent or latent in the donor. E = eperythrozoön; H = haemobartonella.

† *M.* = *Microtus; P.* = *Peromyscus.* ‡ Passages in white mice included.

TABLE 3

THE COURSE OF NATURAL MIXED GRAHAMELLA, HAEMOBARTONELLA AND EPERYTHROZOÖN INFECTIONS IN THE GRAY-BACKED DEER MOUSE FOLLOWING SPLENECTOMY AND OF THE SAME INFECTIONS INDUCED IN THE LOCAL SPECIES OF DEER MOUSE

Animal	Splenectomy	Inoculation	October											November						December						
			8	9	10	11	14	15	18	21	24	28	30	2	4	7	14	19	30	7	14	18	19	23	31
Gray-backed deer mouse*																									
9817	Oct. 4	Grahamella culture Dec. 9		0				0			E	E	E		E	E	H	H	H	H	0	HE		GE	
9818	Oct. 4	None		HEG	HEG	EHG	HG	HG	GH	G	HG†														
9819	Oct. 4	Grahamella culture Dec. 9		0				0			E	E	E			E	E	H	H	H	0	H		HG	
Local deer mouse**																									
9797	Sept. 19	Blood from 9818 Oct. 24	0				0			0		0	H	H	HE		EH	0	H	H	0	0		GH	
		Grahamella culture Dec. 9																							
9800	Sept. 19	Blood from 9819 Oct. 24	0				0			0		0	E	E	E		E	E	0	0	0	G		G	
		Grahamella culture Dec. 9																							
9857	None	Grahamella culture Dec. 9																		0	0	0	0		G

* *Peromyscus maniculatus gracilis.*
** *Peromyscus leucopus novaboracensis.*
† Death.
H haemobartonellosis.
E eperythrozoön infection.
G grahamella infection.
In mixed infection the sequence of the letters indicates the relative preponderance of organisms, the heavier preceding lighter infection.

the blood. This example of "interference" or suppression of one infection by another has received consideration in a separate paper (Tyzzer, 1941). While the haemobartonella in the presence of *E. coccoides* may disappear from the blood, as far as may be determined by microscopic examination, it is by no means wholly eradicated, and following the gradual decrease in the number of eperythrozoa to near the vanishing point, a full-blown haemobartonellosis may suddenly appear weeks or months after inoculation. The eperythrozoön has thus in no way rendered the splenectomized white mouse immune to the haemobartonella. There is a point to be noted in the alternation of these two infections. Although well-established *H. peromysci* infections are promptly suppressed by infecting the animal with *E. coccoides*, it is only after infections of the latter organism have run their natural course and have nearly or actually disappeared that the latent haemobartonella reappears. Thus, the haemobartonella has no pronounced suppressive effect upon the eperythrozoön. In this instance an organism occurring in its natural host interferes with induced infections of an organism from another host species, and may thus wholly prevent the severe anemia and its associated symptoms that result from uncomplicated haemobartonellosis.

The association of eperythrozoön and haemobartonella in the gray-backed mouse has afforded an opportunity to follow the course of double infections in their natural host following splenectomy. Data relating to these animals are presented in Table 3. Unfortunately, one animal, No. 9818, presenting triple infection with grahamella, haemobartonella and eperythrozoön was killed accidentally. In this animal for three successive days the three organisms were well represented in stained blood films, but the eperythrozoön was not apparent for ten days prior to the animal's death. In the other two animals eperythrozoön infection ran a course of at least fourteen and twenty-one days respectively before being replaced by haemobartonella.

Thus, with two infections in the natural host, there seems to be some degree of interference, with the eperythrozoön repressing the haemobartonella, although in the case of the triple infection, mixed infection persisted for several days with eperythrozoön being the first to disappear. To what extent the rather heavy grahamella infection present in this case influenced the other two infections is problematical. In animals in which any one of the three types of infection has run its course and disappeared microscopically from the blood, no immunity to either of the other types of organism has been noted. The inoculation of an animal of another species, *i.e.* the local deer mouse, with the blood from the triple infection resulted in haemobartonellosis followed by mixed haemobartonella and eperythrozoön infection, with the eperythrozoön being the first to disappear. Thus, *E. varians* on injection together with *H. peromysci maniculati* into a foreign host, *e.g.* the local deer mouse, has shown no suppressive effect on the latter organism. Another animal of the same species inoculated with the blood of one of the gray-backed mice at a time when it showed only eperythrozoön, the haemobartonella being latent, developed only eperythrozoön infection. Likewise the eperythrozoön of the white mouse, when transferred to a foreign host, *i.e.* the vole, failed to show any effect upon subsequently induced *Haemobartonella microti* infection. One of four splenectomized voles which were inoculated with *Eperythrozoön coccoides* developed a lasting infection, and another a transient infection. Following the inoculation of all four voles fifteen days later with the vole haemobartonella, a heavy bartonellosis developed promptly in both eperythrozoön-infected voles.

In contrast to the interference effects in connection with associated infections observed in the present study, Neitz (1939) reports that infections with *Babesia canis*, *Babesia gibsoni* and *Rickettsia canis* have an activating effect on latent *Bartonella canis* infection in the dog.

It is possible that the phenomenon of bacterial antagonism found by various investigators (DuBos, 1939—Krasilnikov and Koreniako, 1939—Ark and Hunt, 1941) to be effective in cultures of various organisms may be related in some way to the interference phenomenon observed in mixed infections.

Comparison of Biological Characteristics of Grahamellae, Haemobartonellae and Eperythrozoa

Grahamellae.—In the cultivation of grahamellae from various species of animals on dead media, it is found that they differ appreciably from one another in their growth characteristics. On account of the inability of organisms of this type to multiply on ordinary media and their ready growth in media of high hemoglobin con-

tent, it is difficult to conceive of their multiplying in nature apart from host or vector, although their survival properties in the media in which they have grown is great, *e.g.* a culture of *G. peromysci* remaining alive for more than eleven months at room temperatures. The fact that the circulating blood is the habitat of grahamellae suggests that some blood-sucking arthropod may serve as vector. The consistent lack of motility of grahamellae under cultivation and the absence of spirals, interpreted as detached flagella, set them apart from *Bartonella bacilliformis* of the human being. It now seems clear that the organisms heretofore recognized as characteristic grahamellae should be regarded as bacteria. Furthermore, on account of the peculiar type of infection produced in mammals, characterized by greatly restricted multiplication and situation within erythrotcyes, it seems important that the genus *Grahamella* be retained. As to species of grahamellae, in the past about the only criterion utilized for the differentiation was host of origin, the morphology of organisms in the red cells of different hosts being quite similar. While our present information in regard to infectivity of given grahamellae for various host species is rather fragmentary, notable differences in cultural characteristics have been observed, so that it is perhaps best to continue to designate species for the time being at least on the basis of host of provenance. There is some indication of a considerable degree of host specificity but this matter requires further study employing culture material rather than blood for injection. Thus far the only alien host in which induced infections have become well established is closely related to the natural host. There would appear advantages in regarding grahamellae occurring naturally in such closely related hosts as varieties of a single species. It is not known to what extent the considerable number of grahamella species already described from different hosts will eventually be found to deserve specific rank. Agglutination tests, made possible by isolation in culture of grahamellae from various hosts, may possibly aid in the differentiation of species.

In regard to the relative infectivity of blood showing grahamellae and of such organisms grown in culture, the latter material appears to be more certainly infective, as judged by results in the relatively few animals inoculated, perhaps due to the greater dosage represented. Thus, of six deer mice and four voles inoculated with cultures, all became infected, but of four deer mice and one vole injected with blood showing grahamellae, two deer mice failed to develop infection. That the latter were susceptible was proved by infection following the reinoculation of one with infective blood and of the other with cultured grahamellae.

In experimental grahamella infections, while the incidence of infected red blood corpuscles is relatively high soon after the organisms become microscopically demonstrable in stained blood films, the number of rods per cell at this time is low. Thus in a deer mouse inoculated with grahamella culture, no organisms were found at the end of 6 days, but 8 days after inoculation infected red cells were more numerous than in many natural infections. At this time a count of organisms in twenty-five infected cells showed the number to vary from one to 12 per cell, the average being 3.1. Fifteen days after inoculation the number of infected cells had increased to approximately double the number seen at 8 days, the number of organisms per infected cell varying from one to 18, with an average of 8.2 per cell. In red blood corpuscles showing a single rod, this is often found at the margin which here shows an indentation, as though the organisms had sunk into the substance of the cell. On the first appearance of grahamellae in the blood of the infected animal, isolated extracellular organisms are occasionally encountered, and, while it is possible that these have been liberated from cells crushed in the preparation of the blood film, they may represent organisms occurring free in the plasma. Groups of free grahamellae seen in stained films are obviously derived from crushed cells. Grahamellae liberated from the red cell retain their normal size and shape and never show the flattening and distortion so frequently seen in haemobartonellae and eperythrozoa.

Grahamella-infected red cells have appeared somewhat earlier in voles following inoculation (5, 5, 7, and 14 days) than in the case of *P. leucopus novaboracensis* (7, 8, 10 and 13 days). The grahamella of *P. maniculatus gracilis* appeared in 14 days after inoculation in two animals of the same species and in 9 and 14 days respectively in two local deer mice inoculated. A deer mouse injected with the blood of another carrying both grahamella and haemobartonella showed the former 7 days and the latter 9 days after inoculation. With the haemobartonellosis which becomes apparent on splenectomy of trapped animals carrying haemobartonella and grahamella infection, the latter usually disappears until

the haemobartonellosis has subsided, when it may again be found in the blood films. In several instances, however, the grahamella has persisted even in relatively large numbers throughout the course of rather heavy haemobartonella infections. In some natural infections in which grahamella has not been found previously, it appears on recovery from haemobartonellosis following splenectomy. In regard to the duration of grahamella infection we have no data based on isolations in culture which, as judged by cultures obtained from animals in which the organism could not be demonstrated microscopically, doubtless would be more reliable than the examination of stained blood films. Grahamella has been found in stained blood films from the local deer mouse up to 91 days, and in the vole up to 68 days after inoculation, both animals having been splenectomized. One natural infection in a splenectomized local deer mouse has persisted for 128 days from the time of first observation.

Concerning the effect of splenectomy on grahamellosis, Vassiliadis (1935) found that, while in rodents in general, splenectomy had little effect on this type of infection, in rats the incidence of animals showing microscopically apparent infection was increased from 16.5 per cent before the operation to 93 per cent by splenectomy. In the present series of splenectomies performed on voles, deer mice and shrews infected with grahamella, in only a single deer mouse has a definite increase in the number of organisms been observed. In this case the grahamellae appeared in large numbers preceding and during a light transient bartonellosis brought out by the splenectomy. Vassiliadis attempted to grow grahamella in culture, and, although observing encouraging changes during the first forty-eight hours of incubation, failed to obtain continued growth.

Haemobartonellae.—The effect of splenectomy in bringing about great multiplication of organisms of this type assists in their recognition. While it might be going too far to state that they do not occur within the red cells, their distribution over the surface of the erythrocytes, which is especially obvious with organisms projecting from the margin of these cells, is quite characteristic. Morphology is not always a reliable criterion for distinguishing this type of organism and under certain circumstances it would be difficult to identify a given rod as a haemobartonella or a grahamella, or a given ring as a haemobartonella or an eperythrozoön. Many of the rod forms of *H. peromysci maniculati* are even more slender than those of the grahamellae studied. The pattern assumed in the distribution of individual organisms may assist in identifying the type, *e.g.* the spacing of the rods within the red cell in grahamella versus the concentration of haemobartonellae on some portion of the surface of the red cell. However, the distribution of eperythrozoa on the red cell is in some instances quite similar to that of haemobartonellae, but the former are found in greater numbers free from the red cells and in the early stages of infection show a predilection for reticulocytes, while haemobartonellae are usually confined to mature red cells. Multiplication forms are quite obvious in haemobartonellae but are not readily demonstrable in eperythrozoa. Both types of organism present marked flattening and distortion in ordinary stained blood films in contrast to grahamellae which retain their shape when set free from the red cells.

In view of the author's successful cultivation of grahamellae and his uniform failure in regard to the cultivation of *H. muris*, *H. peromysci*, the variety *maniculati* of the latter, and *H. blarinae*, as well as in recent attempts to cultivate *H. microti*, it may be necessary to reopen the question of the cultivation of rodent haemobartonellae. The possibility of confusion with an unrecognized grahamella or with other blood-inhabiting organisms will require consideration and the production of haemobartonellosis in clean animals by the injection of organisms propagated on artificial media over a considerable period will be essential for proof of cultivation. There is evidence indicating low survival properties in haemobartonellae. Ford and Eliot (1928) report attenuation in *H. muris* in blood kept in the ice-box for twenty-four hours after its withdrawal, while twenty-four hours at room temperature completely destroys the organism. In the present study, blood showing great numbers of *H. microti* on being kept in leptospira medium at room temperature for five days was found to have become non-infective.

The haemobartonellae are usually pathogenic to the host in which they occur on removal of the spleen. When splenectomized animals of other species are employed for susceptibility tests, the haemobartonellae studied have been found to be far less host specific than grahamellae.

Eperythrozoa.—The morphology of *E. coccoides* and that of *Haemobartonella muris*, type species of their respective genera, show markedly con-

trasting features. Nevertheless, a growing acquaintance with additional species of eperythrozoa and haemobartonellae brings to light organisms considered to be of the former type, which have coccoid and possibly rod forms, as, for example, *E. varians*, a new species here described, others that may be regarded as haemobartonellae which present an abundance of ring forms, *e.g. H. canis* and *H. microti*. Certain haemobartonellae such as *H. peromysci* and *H. peromysci maniculati* may present notable morphological transformation with excess of rings or coccoids when transferred to certain foreign hosts. Such morphological intergradation can hardly be ascribed to mixed infection for, with the haemobartonellae, the rings appear quite definitely developing within rods. The effects of splenectomy upon latent infections of the haemobartonellae and eperythrozoa, their distribution with relation to the red blood cells, response to arsphenamine treatment, difficulty of cultivation, constitute characteristics common to both types of organism and suggest close phylogenetic relationship.

Summary

A comparative experimental and morphological study has been made of the grahamellae occurring in the short-tailed meadow mouse or vole, the local species of deer mouse, the gray-backed deer mouse, the short-tailed shrew, the pine mouse and a species of Peruvian mouse; the haemobartonellae of the local deer mouse, the gray-backed deer mouse, the short-tailed shrew, and the gray squirrel; and the eperythrozoa of the common albino mouse, the vole and the gray-backed deer mouse. Blood films of *Grahamella talpae*, *Haemobartonella muris* and *H. canis* have also been studied.

New species of *Grahamella*, *Haemobartonella* and *Eperythrozoön* have been described from certain small mammals. The species of grahamella described are differentiated chiefly on the basis of host of origin, but supported by differences observed in cultures isolated from the blood of various host species. Tests indicating a rather marked degree of host specificity in regard to grahamella infections are probably inconclusive except when cultures rather than blood are employed for the test injections. Further study should be given to the question of host specificity as a criterion of grahamella species. The species of haemobartonellae listed are based on host of origin, infectivity and pathogenicity for splenectomized animals of other species, and morphology. The eperythrozoön species are likewise differentiated on host of origin and morphology, with some tests as to infectivity for alien hosts.

Culture strains of grahamellae have been isolated from the vole, the local deer mouse and the gray-backed deer mouse. Koch's postulates in regard to the production of infection by the inoculation of culture material and the recovery of the organism from the inoculated animal have been fulfilled with a culture strain from each of the three species. An organism isolated in culture from the blood of the short-tailed shrew appears to be a grahamella, although its identity is not established by test inoculations. A comparison of the grahamellae isolated in culture, with *Actinomyces muris* (*Streptobacillus moniliformis*) indicates close relationship and the inclusion of the genus *Grahamella* in the family ACTINOMYCETACEAE is proposed.

Numerous attempts to cultivate the haemobartonellae of the vole, two species of deer mice, the short-tailed shrew and the white rat have been uniformly unsuccessful.

Tests have been made as to the infectivity, of the haemobartonellae studied, for different species of animals. While certain haemobartonellae may be infective and pathogenic for splenectomized alien hosts, *H. peromysci* apparently persists for only a brief period in the non-splenectomized white mouse. The transference of haemobartonellae to certain alien hosts may result in profound morphological modification, and even in instances in which the inoculated host is closely related to the donor, notable transformations may occur.

Morphological studies of the haemobartonellae appearing after splenectomy in the blood of the local deer mouse, the gray-backed deer mouse and the short-tailed shrew have revealed features characteristic of higher fungi, *i.e.* the development of branching filaments from spore-like granules, with the production in some cases of rudimentary mycelia and multiplication by the formation of bead-like elements within the filament as well as by segmentation. These characters are not apparent in the type species, *Haemobartonella muris* Mayer, 1921, which shows no notable tendency to form filaments in the blood of the rat. *H. canis* Kikuth, 1928, on the other hand, presents a moderate degree of branching.

LIST OF GRAHAMELLAE, HAEMOBARTONELLAE AND EPERYTHROZOA STUDIED

Organism*	Host**	Comment
G. peromysci, n. sp.	*P. leucopus novaboracensis*	Grown in culture.
G. peromysci maniculati, n. var.	*P. maniculatus gracilis*	Grown in culture. Infective for *P. leucopus novaboracensis.*
G. microti pennsylvanici, n. var.	*M. pennsylvanicus pennsylvanicus*	Grown in culture.
G. blarinae, n. sp.	*Blarina brevicauda*	Culture obtained but not tested by inoculation.
G. phyllotidis, n. sp.	*Phyllotis darwini linatus*	
G. sp.?	*Pitymys pinetorum*	
H. microti Tyzzer and Weinman, 1939	*M. pennsylvanicus pennsylvanicus*	Infective for splenectomized white mice.
H. peromysci, n. sp.	*P. leucopus novaboracensis*	Infective and pathogenic for splenectomized white mice, rats and voles.
H. peromysci maniculati, n. var.	*P. maniculatus gracilis*	Non-infective for splenectomized white mice. Infective for *P. leucopus novaboracensis.*
H. blarinae, n. sp.	*Blarina brevicauda*	Non-infective for splenectomized white mice.
H. sciurae, n. sp.	*Sciurus carolinensis leucotis*	Non-infective for non-splenectomized white mice.
E. coccoides Schilling, 1928	*Mus decumanus albus*	Slightly infective for splenectomized voles.
E. dispar Bruynoghe and Vassiliadis, 1929	*Arvicola (Microtus) arvalis*†	Non-infective for splenectomized white mice (Tyzzer and Weinman, 1939).
	M. pennsylvanicus pennsylvanicus	Infective for *P. leucopus novaboracensis.*
E. varians, n. sp.	*P. maniculatus gracilis*	Non-infective for splenectomized white mice.

* *G.* = *Grahamella; H.* = *Haemobartonella; E.* = *Eperythrozoön.*
** *P.* = *Peromyscus; M.* = *Microtus.*
† Species probably the same in both the European and the American vole.

Nevertheless, there would not as yet appear to be sufficient basis for the inclusion of the haemobartonellae with the higher fungi, and further classification is left until the matter of cultivation is thoroughly explored. Notwithstanding superficial resemblances of haemobartonellae to *Bartonella bacilliformis* as seen in stained blood films, it appears that there is no close phylogenetic relationship.

The injection of eperythrozoa into splenectomized hosts closely related to the host of origin has produced enduring infections but in less closely related hosts only transient infections.

Interference has been observed in connection with associated eperythrozoön and haemobartonella infections in the host in which the eperythrozoön is indigenous. The suppression of one type of infection by another is not based on immunity in the ordinary sense, for with the disappearance of the dominating organism the suppressed organism reappears. The suppressive effect in such cases may be as complete for a period as that derived from the spleen. When eperythrozoön and haemobartonella infections are transferred to a foreign host, the former has no suppressive effect upon the latter.

The isolation in culture of several strains of filament-forming organisms from the blood of apparently normal voles is indicative of the prevalence of inapparent blood infections in this rodent in addition to the named species of organisms discussed. From the moniliform character of the filaments and from the presence of globoid bodies in great variety, the above strains resemble *Actinomyces muris* but rarely cause death when injected into mice. The study of the blood infections of small mammals emphasizes the frequency of infections maintained in nature, so perfectly adjusted to the host in which they occur as to be without appreciable injurious effect.

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THEORETICAL LIGHT CURVES OF CLOSE ECLIPSING SYSTEMS

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ABSTRACT

The aim of the present study will be to investigate the theoretical light curves of close eclipsing systems, between eclipses as well as within minima, taking account of the departures of both components from spherical shape due to their axial rotation and mutual tidal action. The distortion influences the light curves both directly, by varying the area visible at various phases, and indirectly, by rearranging the distribution of brightness over the distorted surface which, in addition to limb-darkening, should vary proportionally to the local gravity. The form of each component is supposed to be one of equilibrium under instantaneous rotational and tidal forces, and predictable theoretically so long as the axial rotation is slow and the tidal action of each star can be regarded as being due to a mass-point.

In the present paper photometric consequences of distortion of both components of an eclipsing binary will be investigated to the same degree of accuracy to which their own forms are known (*i.e.*, as far as effects of the second, third, and fourth spherical harmonic deformations are concerned). Account will be taken of the combined effects of geometrical boundary distortion as well as of limb-darkening and gravity-darkening at any particular frequency of light. The components will be supposed to describe circular orbits; no notice will be taken of complications that would arise from orbital eccentricity. After a brief general survey (sec. I), equations of the problem are set up (sec. II) and reduced to tractable forms (sec. III). Photometric implications of the gravity-darkening will prove relatively most troublesome and a special class of functions will be introduced (sec. IV) to deal efficiently with this particular part of the problem. The explicit forms of the theoretical light curves are then given in section V in terms of certain integrals which are subsequently evaluated in a finite number of terms (sec. VI). The concluding section VII will be devoted to a more general discussion concerning the "rectification" of light curves and the application of the preceding results to practical cases. It is pointed out that assumptions conventionally associated with the process of rectification are not generally fulfilled, and more appropriate procedures are developed.

The results of the present paper should, apart from the reflection effect, give a complete representation of the theoretically expected changes of light of close eclipsing systems between minima as well as within eclipses—regardless of whether the star undergoing eclipse is the larger or the smaller of the two and whether the eclipse is partial or annular.

I. INTRODUCTION

1. A general task of determining the elements of eclipsing binary systems from an analysis of their light curves necessarily requires a knowledge of the loss of light, during eclipse, for any relative position of the components. This evidently depends on the form of both components as well as on the distribution of brightness over the disk of the eclipsed star. For spherical components with uniform or radially-symmetrical distribution of brightness the relation between the geometry of eclipse and the corresponding loss of light is well known. The reality, however, is likely to be more complex. Unless the stars were rigid, the components of binary systems can be regarded as approximately spherical only if their dimensions, expressed in terms of their separation, are very small. If, however, the components are brought closer together, their surfaces will deviate from a sphere, on account of

(*a*) the rotational distortion,
(*b*) the mutual tidal action.

Provided that stars behave as fluid bodies whose periods of free oscillation are short compared with the period of the binary orbit,[1] the tidal distortion of both components will be given by the equilibrium theory of tides and is at present accurately predictable as far as the disturbing action of one component upon the other can be regarded as that of a mass-point. The rotational distortion can be predicted to the same degree of accuracy provided that the axes of rotation are constantly perpendicular to the orbital plane. In the case of components revolving in circular orbits we may legitimately assume that their surface layers rotate with the Keplerian angular velocity; whether or not their interiors rotate in this way is largely irrelevant as far as the external form of the components is concerned.[2]

The problem which we propose to attack and solve in the present investigation can be briefly stated as follows: *Let us consider a close binary system, the components of which describe circular orbits in a plane inclined to the line of sight so that the components eclipse each other alternately at the time of conjunction. Both stars are distorted by*

[1] *Cf. e.g.*, Sterne, *M.N.*, **99**, 662, 1939.
[2] *Cf.* Kopal, *Proc. Nat. Acad. Sci.*, **27**, 359, 1941.

axial rotation with Keplerian angular velocity and by mutual tidal action. The outward flux of radiant energy at the surface of the distorted components varies proportionally to the local gravity.[3] *It is to investigate the changes of light exhibited by such systems in any particular frequency of light, and to the same degree of accuracy to which the form of both components is known.* The changes of light displayed by such systems between minima have recently become a subject of numerous papers;[4] but changes of light within eclipses—a geometrically much more complicated phenomenon—are as yet but very incompletely known and remain to be investigated. In the present paper such an investigation will be made.

The effects, upon light curves, of the distortion of both components can be regarded as consisting of two parts:

(1) effects arising from the distortion of the eclipsing component,
(2) effects due to the distoriton of the eclipsed star.

Effects of the former kind act through the distortion of the shadow cylinder cast by the eclipsing body, and have previously been studied under certain restricted conditions by Walter[5] and Russell,[6] and more generally by Takeda[7] and Kopal.[8] They are geometrical in nature and relatively simple. On the other hand, effects mentioned in (2) involve also physical considerations (law of radiation) and are vastly more complex. This is because in dealing with photometric effects of distortion of the star undergoing eclipse—which will consistently be referred to as the *primary component*—we have not only to take account of the distorted geometry, but also its effects upon the distribution of brightness over the eclipsed portion of the primary's disk. For it is well known that the distribution of brightness can be regarded as uniform or radially-symmetrical (limb-darkening) only if a star is spherical or very nearly so; but if it gets appreciably distorted, the distribution of brightness over its surface should, in addition to limb-darkening, vary proportionally to the local gravity.[3] Recent investigations of the ellipticity effect in close eclipsing systems[9] have established the reality of the gravity-effect upon the distribution of surface brightness in distorted stars beyond any doubt, and in the amount theoretically expected. It therefore appears that no substantial advance in our knowledge of close binary systems can be made unless the bearing of the gravity-effect upon the light curves within minima is investigated in detail. This sets the task which we are going to face.

Its inherent complexity has probably been discouraging for most investigators so far, and the only existing attempt at its solution appears to be that by Takeda.[7] The starting points of Takeda's and of the present paper will necessarily have many features in common; in particular, the transformation of coördinates which we shall employ in section III of the present paper has also been used by Takeda. The present investigation will, however, be considerably more general in scope. Takeda's aim has been to study variation of integrated light of eclipsing systems with components built up according to the standard model (in point of fact, Takeda's coefficients pertaining to the tidal distortion correspond to a mass-point model). A centrally condensed model offers probably an ample approximation for most of the real eclipsing systems. As will be shown later in more detail, however, the restriction to bolometric light changes is decidedly inadequate and would render Takeda's results of little value for application to actual light curves observed in more or less narrow spectral ranges. In addition, Takeda did not evaluate the forms of the theoretical light curves explicitly, but expressed them in terms of certain integrals the solution of which was largely left as an exercise to the reader. In the present paper our aim will be to evaluate in a finite number of terms the changes of light due to the eclipses of distorted stars of any internal structure, and observed in any particular frequency of light; and the analysis will include effects of the second, third, and fourth spherical harmonic deformations of both components due to their own rotation and mutual tidal action.

II. Equations of the Problem

2. The loss of light, during eclipse, as seen by an observer at a great distance, can generally be

[3] *Cf.* von Zeipel, *M.N.*, **84**, 665, 684, 702, 1924; and Chandrasekhar, *M.N.*, **93**, 571–573, 1933.

[4] Takeda, *Kyoto Mem.*, A, **18**, 197, 1934; Russell, *Ap.J.*, **90**, 641, 1939; Kopal, *Ann. New York Acad. Sci.*, **41**, 13, 1941; Sterne, *Proc. Nat. Acad. Sci.*, **27**, 99, 1941.

[5] *Königsberg Veröff.*, No. 2, p. 28, 1931.

[6] *Ap.J.*, **90**, 641, 1939.

[7] *Kyoto Mem.*, A, **20**, 47, 1937.

[8] *Ap.J.*, **94**, 159, 1941.

[9] Russell, *Ap.J.*, **90**, 641, 1939; or Kopal, *Ann. New York Acad. Sci.*, **41**, 13, 1941.

expressed as

$$\Delta\mathfrak{L} = \int_S J \cos\gamma \, d\sigma, \tag{1}$$

where J denotes the intensity at any point of the surface of the eclipsed star; γ, the angle of foreshortening; and the surface element $d\sigma$ is evidently given by

$$\cos\beta \, d\sigma = r^2 \sin\theta \, d\theta \, d\phi;$$

θ, ϕ being polar coördinates taken with respect to a rectangular system the X-axis of which is constantly coincident with the line joining the centers of both components, and Z with that of rotation of the primary star (the center of which is taken as origin). The angle β is one between a radius-vector and a normal to the surface $r(\theta, \phi)$ specifying the boundary of the distorted primary star. The range of integration S is to be extended over the whole eclipsed area (for which $\cos\gamma$ is positive).

The angles β and γ are clearly defined by

$$\cos\beta = \lambda l + \mu m + \nu n, \tag{2}$$

$$\cos\gamma = l l_0 + m m_0 + n n_0, \tag{3}$$

where

$$\begin{array}{llll} \lambda, & \mu, & \nu \\ l, & m, & n \\ l_0, & m_0, & n_0 \end{array}$$

denote direction cosines of a radius-vector, surface normal, and line of sight, respectively.

The values of λ, μ, ν and l_0, m_0, n_0 depend upon the adopted frame of reference and the observer's position. In conformity with the preceding we have

$$\begin{aligned} \lambda &= \cos\phi \sin\theta, \\ \mu &= \sin\phi \sin\theta, \\ \nu &= \cos\theta, \end{aligned} \tag{4}$$

where ϕ denotes the angle, in the XY plane, between the projected radius-vector and the X-axis; and θ is the angle between a radius-vector and the Z-axis. Similarly,

$$\begin{aligned} l_0 &= \cos\psi \sin i, \\ m_0 &= \sin\psi \sin i, \\ n_0 &= \cos i, \end{aligned} \tag{5}$$

where ψ denotes the mean anomaly in the plane of the orbit (phase angle), and i, the inclination of the orbital plane to the celestial sphere.

3. The values of the remaining direction cosines l, m, n depend on the form of the free surface of the distorted primary star. If the latter is in equilibrium under the forces exerted upon it by its own rotation and by the tidal pull of the eclipsing companion, its form can at present be predicted as far as the disturbing action of the companion can be regarded as that of a mass-point. Further, if the primary component rotates about a fixed axis perpendicular to the orbital plane with Keplerian angular velocity, the rotational distortion—to this order of accuracy—takes the form

$$-\frac{1}{3}\left(1 + \frac{m_2}{m_1}\right) \Delta_2 r_1^3 P_2(\nu), \tag{6}$$

and the tidal distortion becomes

$$+\frac{m_2}{m_1} \sum_{j=2}^{4} \Delta_j r_1^{j+1} P_j(\lambda), \tag{7}$$

where m_2/m_1 denotes the mass-ratio; r_1, the radius of a sphere having the same volume as the distorted primary star and expressed in terms of the radius of circular orbit; and the P_j's are Legendre polynomials of various orders. The Δ_j's are numerical constants depending on the internal structure (density distribution) of the distorted star and are generally defined by

$$\Delta_j = \frac{2j+1}{j + \eta_j(r_1)}, \tag{8}$$

where $\eta(r_1)$ is the surface value of a function defined by the differential equation

$$r\frac{d\eta_j}{dr} + \eta_j(\eta_j - 1) + 6\frac{\rho}{\bar{\rho}}(\eta_j + 1) = j(j+1),$$

which is due to Radau.[10] Here ρ denotes the density at any point in the interior of the distorted star while $\bar{\rho}$ is the mean density interior to r.

The numerical values of the Δ_j's depend in principle on the whole march of density distribution. In practice, however, they turn out to be almost uniquely specified by the ratio of the central to the mean density (density condensation) of a given configuration.[11] Except for a few special cases Radau's equation does not admit of a solution in a closed form and has to be integrated numerically. The most complete integrations are so far available for the polytropic family of models,[12] and the results are compiled in the following Table 1.

[10] *Comptes Rendus*, **100**, 972, 1885; *cf.* also Tisserand, *Traité de Mécanique Céleste*, Tome II, p. 221, Paris 1891. Radau derived his equation for the special case of $j = 2$.

[11] *Cf.* Kopal, *M.N.*, **98**, 414, 589, 1938; or Sterne, *M.N.*, **99**, 662, 1939 (p. 664, footnote).

[12] Chandrasekhar, *M.N.*, **93**, 449, 1933 (Table VI). Chandrasekhar's results were obtained by numerical integration of less general equations.

TABLE 1

Polytropic Index n	0	1	1.5	2	3	4	5
ρ_c/ρ_m	1	3.28987	5.9907	11.4025	54.1825	622.408	∞
$\eta_2(r_1)$	0	1.28986	1.8784	2.3546	2.8596	2.98669	3
Δ_2	2.5	1.51982	1.2892	1.1482	1.0289	1.00267	1
$\eta_3(r_1)$	1	2.77124	3.3183	3.6743	3.94887	3.99672	4
Δ_3	1.75	1.21291	1.1079	1.0488	1.00736	1.00047	1
$\eta_4(r_1)$	2	4.03228	4.5211	4.7968	4.97480	4.99874	5
Δ_4	1.5	1.12048	1.0562	1.0231	1.00281	1.00014	1

The direction cosines of a line normal to the distorted surface specified by (6) and (7) then are

$$\begin{aligned} l &= \lambda\left\{1-v_1^{(2)}\nu^2+\left(\lambda-\frac{1}{\lambda}\right)\sum_{j=2}^{4}w_1^{(j)}P_j'(\lambda)\right\},\\ m &= \mu\left\{1-v_1^{(2)}\nu^2 \quad + \quad \lambda\sum_{j=2}^{4}w_1^{(j)}P_j'(\lambda)\right\}, \qquad (9)\\ n &= \nu\left\{1-v_1^{(2)}\nu^2+v_1^{(2)}+\lambda\sum_{j=2}^{4}w_1^{(j)}P_j'(\lambda)\right\}, \end{aligned}$$

where accents on P_{j-1} denote derivatives with respect to λ, and where we have abbreviated [13]

$$\begin{aligned} v_{1,2}^{(2)} &= \left(1+\frac{m_{2,1}}{m_{1,2}}\right)\Delta_2 r^3_{1,2},\\ w_{1,2}^{(j)} &= \frac{m_{2,1}}{m_{1,2}}\Delta_j r_{1,2}^{j+2}. \qquad (10.0) \end{aligned}$$

Both v and w are evidently small quantities of the first order which specify the amount of distortion. As the reader may easily verify the angle β between a radius-vector and surface normal is of the same order of magnitude as the distortion itself; *i.e.*, a small quantity of the first order. Hence, within the scheme of our approximation, the squares and higher powers of β can be neglected and $\cos\beta$ reduces to unity. The angle of foreshortening γ is then obtained from (3) as

$$\begin{aligned} \cos\gamma = L\Big\{1 &+ \tfrac{1}{3}v_1^{(2)}\left[\frac{n_0}{L}-\nu\right]P_2'(\nu)\\ &-\left[\frac{l_0}{L}-\lambda\right]\sum_{j=2}^{4}w_1^{(j)}P_j'(\lambda)\Big\}, \qquad (3.1) \end{aligned}$$

[13] If the eclipsed primary star does not rotate with the Keplerian angular velocity (which is unlikely for systems revolving in circular orbits), but with an independent angular velocity ω, then

$$v_1^{(2)} = \tfrac{5}{2}\Delta_2\frac{\omega^2}{2\pi G\rho_m},$$

where G is the gravitation constant and ρ_m the mean density of the rotating star.

where

$$L = \lambda l_0 + \mu m_0 + \nu n_0.$$

4. The expression for J remains to be specified. The effects of darkening at limb should, with sufficient accuracy, be satisfied by the well-known cosine law

$$J = H(1 - u + u\cos\gamma),$$

where H denotes the intensity of radiation parallel to the line of sight and u the coefficient of limb darkening.

As regards H, a theorem was proved by von Zeipel [14] that the outward flux of total radiation (*i.e.*, bolometric surface brightness) over the surface of a distorted star in hydrostatic equilibrium must be proportional to the local gravity. The proof of the theorem presumes that stars rotate with uniform angular velocity and that there are no appreciable convection currents beneath the photosphere. It does not seem likely that deviations from uniform rotation could materially change the theorem; but an appreciable transport of heat by material currents may make the flow of heat no longer proportional to the temperature gradient and so render the theorem invalid. It is difficult to predict solely on the theoretical basis to which extent von Zeipel's theorem should apply to actual stars; but its consequences upon changes of light exhibited by close eclipsing systems between minima (ellipticity effect) were recently found exactly in agreement with the observed facts.[9] Hence the application of the theorem to reality seems fully justified and we have to put

$$\frac{H - H_0}{H_0} = \frac{g - g_0}{g_0}, \qquad (12)$$

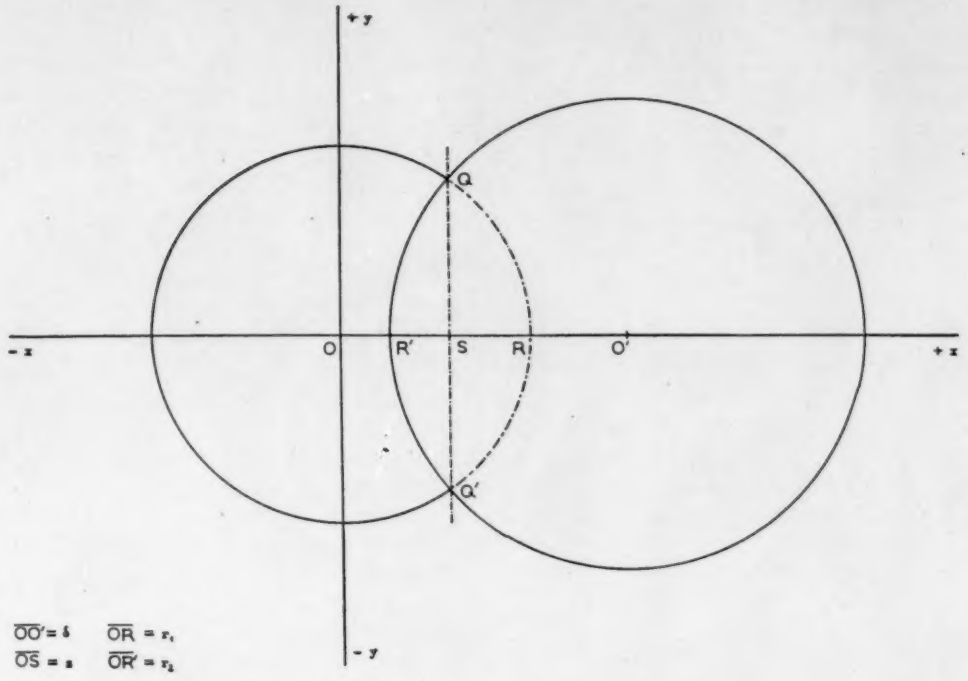
where g and g_0 denote the local and mean surface gravity, respectively, and H_0 is the bolometric surface brightness at a point where $g = g_0$.

If the star undergoing eclipse radiates like a black body, equation (12) can evidently be rewritten as

$$\frac{g - g_0}{g_0} = \frac{T^4 - T_0^4}{T_0^4}, \qquad (13)$$

where T denotes the local effective temperature and T_0 the mean effective temperature averaged over the whole disk. Equation (13) governs the variation of temperature over the surface of a

[14] *M.N.*, **84**, 665, 684, 702, 1924. *Cf.* also Chandrasekhar, *M.N.*, **93**, 539, 1933 (Appendix I); or Sterne, *Proc. Nat. Acad. Sci.*, **27**, 99, 1941.



distorted star. The surface brightness at any single wave length λ varies as

$$\frac{H}{H_0} = \frac{e^{c_2/\lambda T_0} - 1}{e^{c_2/\lambda T} - 1}, \tag{14.0}$$

where $c_2 = 1.432$ cm.deg. If we substitute in (14.0) for T from (13) and expand the former in a Taylor series in the neighborhood of $T = T_0$ then, to the first order in small quantities, equation (14.0) reduces to

$$\frac{H}{H_0} = 1 - \tau\left(1 - \frac{g}{g_0}\right), \tag{14.1}$$

where

$$\tau = \frac{c_2}{4\lambda T_0(1 - e^{-c_2/\lambda T_0})}. \tag{15}$$

If $\tau = 1$ (*i.e.*, $\lambda T_0 = 0.3652$ cm.deg.) equation (14.1) gives the intensity distribution for total radiation. In practical cases, however, τ may be either greater or smaller than unity. A glance at (15) shows that if $(c_2/\lambda T_0)$ is small (high temperatures or long wave lengths) τ converges to one fourth; while if $(c_2/\lambda T_0)$ is large, τ varies proportionally to it. A three-decimal table of 4τ in terms of λ and T_0 has recently been prepared by the writer.[15]

[15] *Ann. New York Acad. Sci.*, **41**, 13, 1941 (Table 2).

The surface gravity of a star the distortion of which is specified by (6) and (7) can be shown[16] to vary as

$$\frac{g - g_0}{g_0} = \tfrac{1}{3}v_1^{(2)}\left\{\frac{5}{\Delta_2} - 1\right\}P_2(\nu) - \sum_{j=2}^{4}\left\{\frac{2j+1}{\Delta_j} - j + 1\right\}w_1^{(j)}P_j(\lambda). \tag{16}$$

Inserting this in (14.1) we obtain the proper distribution of brightness due to the gravity effect.

5. The fractional loss of light $\Delta\mathfrak{L}$ within minima can evidently be expressed as

$$\Delta\mathfrak{L} = \frac{3(1-u)}{3-u}\Delta\mathfrak{L}^U + \frac{2u}{3-u}\Delta\mathfrak{L}^D, \tag{17.0}$$

where $\Delta\mathfrak{L}^U$ and $\Delta\mathfrak{L}^D$ denote the "uniform" and "completely darkened" fractional losses of light defined by

$$\pi r_1^2 H_0 \Delta\mathfrak{L}^U = \int_S r^2 H \cos\gamma \sin\theta\, d\theta\, d\phi, \tag{18.0}$$

and

$$\pi r_1^2 H_0 \Delta\mathfrak{L}^D = \frac{3}{2}\int_S r^2 H \cos^2\gamma \sin\theta\, d\theta\, d\phi. \tag{18.1}$$

If we now combine equations (3.1), (6), (7), (14.1) and (16) the integrand of (18.0) takes the

[16] *Cf.* Çhandrasekhar, *M.N.*, **93**, 539, 1933 (Equation 123).

neat form

$$r^2H\cos\gamma = r_1^2H_0L\left\{1+\tfrac{1}{3}v_1^{(2)}\left[\left\{\tau\left(\frac{5}{\Delta_2}-1\right)-2\right\}P_2(\nu)+\left\{\frac{n_0}{L}-\nu\right\}P_2'(\nu)\right]\right.$$
$$\left.-\sum_{j=2}^{4}w_1^{(j)}\left[\left\{\tau\left(\frac{2j+1}{\Delta_j}+1-j\right)-2\right\}P_j(\lambda)+\left\{\frac{l_0}{L}-\lambda\right\}P_j'(\lambda)\right]\right\}, \quad (19.0)$$

while for darkened disks

$$r^2H\cos^2\gamma = r_1^2H_0L^2\left\{1+\tfrac{1}{3}v_1^{(2)}\left[\left\{\tau\left(\frac{5}{\Delta_2}-1\right)-2\right\}P_2(\nu)+2\left\{\frac{n_0}{L}-\nu\right\}P_2'(\nu)\right]\right.$$
$$\left.-\sum_{j=2}^{4}w_1^{(j)}\left[\left\{\tau\left(\frac{2j+1}{\Delta_j}+1-j\right)-2\right\}P_j(\lambda)+2\left\{\frac{l_0}{L}-\lambda\right\}P_j'(\lambda)\right]\right\}. \quad (19.1)$$

If we consider effects of total radiation only, and assume the distorted star to be a centrally condensed one, the $\tau = 1$ and an inspection of Table 1 reveals that the Δ_j's may be put equal to unity without any appreciable error. Making use of the identity

$$\lambda P_j'(\lambda) = jP_j(\lambda) + P'_{j-1}(\lambda)$$

the right-hand side of (19.0) reduces to

$$r_1^2H_0L\left\{1+\tfrac{1}{3}v_1^{(2)}\left[\frac{n_0}{L}P_2'(\nu)-1\right]\right.$$
$$\left.-\sum_{j=2}^{4}w_1^{(j)}\left[\frac{l_0}{L}P_j'(\lambda)-P'_{j-1}(\lambda)\right]\right\}, \quad (19.01)$$

and that of (19.1) can be simplified in an analogous way. To this degree of approximation the problem has been dealt with by Takeda.[4] We note, however, that while an assumption $\Delta_j \approx 1$ offers a satisfactory approximation in the majority of practical cases, τ may happen to be considerably in excess of, or smaller than, unity; so that $|\tau - 1|$ cannot in general be regarded a small quantity. Hence the scheme of approximation adopted by Takeda appears inadequate, and more complete expressions (19.0) and (19.1) with an appropriate value of τ are to be employed.

III. Transformation of Coördinates

6. In the preceding section the integrands of (18.0) and (18.1) have been expressed in terms of polar coördinates referred to a fixed rectangular frame XYZ, with origin at the center of the eclipsed star, and axes parallel with the three principal axes of the distorted ellipsoid. In order to be able to carry out the integration we still have to specify the limits. In doing so, we soon discover that a fixed "astrocentric" system of coördinates is unsuitable for the study of light changes within minima; the limits result too complicated and the respective integrals become almost intractable. As will be shown below, however, this difficulty can be avoided by a suitable change of coördinates.

Let us change over to a moving rectangular system $X'Y'Z'$ having the same origin, but with the X'-axis now constantly coincident with the line of sight and the Z'-axis in the direction of the projected center of the secondary component. The new primed axes, referred to the original system, are then clearly specified by the scheme

	X'	Y'	Z'
X	l_0	l_1	l_2
Y	m_0	m_1	m_2
Z	n_0	n_1	n_2

where the direction cosines of the respective axes are:

$$l_0 = \cos\psi\sin i, \qquad l_1 = 0,$$
$$l_2 = \sqrt{1-l_0^2},$$

$$m_0 = \sin\psi\sin i, \qquad m_1 = -\frac{n_0}{\sqrt{1-l_0^2}},$$
$$m_2 = -\frac{l_0m_0}{\sqrt{1-l_0^2}}, \quad (20)$$

$$n_0 = \cos i, \qquad n_1 = \frac{m_0}{\sqrt{1-l_0^2}},$$
$$n_2 = -\frac{l_0n_0}{\sqrt{1-l_0^2}}.$$

The coördinates of the projected center of the eclipsing component before maximum eclipse are assumed to be positive.

The eclipsed portion of the primary's disk can now be specified as follows. The equation of the shadow cylinder cast by the secondary component in the direction of the line of sight in the primed system becomes

$$(\delta - Z')^2 + Y'^2 = r_2^2(1 + \Delta\bar{r}_2)^2, \qquad (21)$$

where r_2 is the fractional radius of a sphere having the same volume as the eclipsing star; $\Delta\bar{r}_2$ is a measure of deformation of the secondary's shadow cylinder in the $Y'Z'$-plane; and δ, the projected distance of centers of the two components in the same plane is clearly given by

$$\delta^2 = \sin^2\psi \sin^2 i + \cos^2 i.$$

The radius of the orbit is taken as our unity of length.

The equation of surface of the distorted primary star admits evidently of the parametric representation

$$\begin{aligned} X' &= r_1(1 + \Delta r_1)L, \\ Y' &= r_1(1 + \Delta r_1)M, \\ Z' &= r_1(1 + \Delta r_1)N, \end{aligned} \qquad (22)$$

where Δr_1 is a measure of distortion of the primary component, and L, M, N are direction cosines of any radius-vector in the primed system, related with the direction cosines λ, μ, ν of the fixed system by means of

$$\begin{aligned} \lambda &= l_0L + l_1M + l_2N, \\ \nu &= n_0L + n_1M + n_2N. \end{aligned} \qquad (23)$$

The direction cosines L, M, N can evidently be regarded as rectangular coördinates in the primed system over a sphere of unit radius. We prefer, however, consistently with the preceding, to adopt the radius of the orbit as our unity of length. Let us therefore put

$$L = \frac{z}{r_1}, \quad M = \frac{y}{r_1}, \quad N = \frac{x}{r_1}, \qquad (24)$$

and adopt x, y, z as our new independent variables. By virtue of (23) the Legendre coefficients $P_2(\nu)$ or $P_j(\lambda)$, associated with effects of the rotational and tidal distortion, can now be rewritten as polynomials of the j-th degree in terms of integral powers of x, y, z, with coefficients depending upon the amount of distortion and the relative position of the components in their orbit. Since, further,

$$\sin\theta\, d\theta\, d\phi = \frac{dxdy}{r_1\sqrt{r_1^2 - x^2 - y^2}},$$

the whole integrand of (18.0) or (18.1) in terms of the new variables clearly becomes an algebraic function of x, y, and z—the general terms arising from the rotational and tidal distortion being of the form

$$x^m y z^n \quad \text{and} \quad x^m z^n,$$

respectively, where $m \geqslant 0$ and $n \geqslant -1$.

7. The range of integration can now be found as follows. The direction cosines L, M, N are known to satisfy the relation

$$L^2 + M^2 + N^2 = 1.$$

It may be remembered that L, the cosine of the angle of foreshortening, vanishes at the limb of the primary's disk if the latter is undistorted, and becomes a small quantity in the case of distortion, the square of which can be neglected. Hence, to the order of accuracy we are working, the intersection of the surface of the primary component with the xy-plane reduces to

$$x^2 + y^2 = r_1^2, \qquad (25)$$

and thus the arc QRQ' limiting the eclipsed fraction in the xy-plane (see Fig. 1) can be considered a circle.

The equation of the remaining arc $QR'Q'$ can be obtained by solving the equation of the shadow cylinder with that of the surface of the eclipsed star. If we insert (22) in (21) then, to the first order in small quantities, the equation of $QR'Q'$ becomes

$$\begin{aligned} (\delta - x)^2 + y^2 = r_2^2(1 + 2\Delta\bar{r}_2) \\ - 2\Delta r_1\{r_2^2 - \delta(\delta - x)\} + \cdots. \end{aligned} \qquad (26)$$

Now, to the same order of approximation, expressions (6) and (7) yield

$$\Delta r_1 = \sum_{j=2}^{4} w_1^{(j)} P_j(\lambda) - \tfrac{1}{3} v_1^{(2)} P_2(\nu), \qquad (27)$$

where

$$\begin{aligned} r_1\lambda &= l_0\sqrt{2\delta(s - x)} + l_2x, \\ r_1\nu &= n_0\sqrt{2\delta(s - x)} \\ &\quad + n_1\sqrt{r_2^2 - (\delta - x)^2} + n_2x, \end{aligned} \qquad (28)$$

and

$$s = \frac{r_1^2 - r_2^2 + \delta^2}{2\delta}. \qquad (29)$$

Further, the corresponding distortion of the shadow cylinder cast by the secondary component takes the form

$$\Delta\bar{r}_2 = \sum_{j=2}^{4} w_2^{(j)} P_j(\lambda') - \tfrac{1}{3} v_2^{(j)} P_2(\nu'), \qquad (30)$$

where, since the line of sight is tangent to the boundary of the shadow cylinder,

$$r_2\lambda' = l_2(\delta - x),$$
$$r_2\nu' = n_1\sqrt{r_2^2 - (\delta - x)^2} + n_2(\delta - x). \tag{31}$$

The deviation, in the y-coördinate, of arc $QR'Q'$ from the circle

$$(\delta - x)^2 + y^2 = r_2^2 \tag{32}$$

then results

$$\Delta y = \frac{r_2^2\Delta r_2}{\sqrt{r_2^2 - (\delta - x)^2}} - \{r_2^2 - \delta(\delta - x)\}\frac{\Delta r_1}{\sqrt{r_2^2 - (\delta - x)^2}} + \cdots. \tag{33}$$

8. The limits of integration specifying the eclipsed portion of the primary's disk then result as follows. Let us divide the total eclipsed area in two parts: $QRQ'S$ and $QR'Q'S$ (Fig. 1). The light lost by the eclipse of $QRQ'S$ does not depend explicitly upon the radius of the eclipsing star and can be obtained by integrating (18.0) or (18.1) between $\pm\sqrt{r_1^2 - x^2}$ in the y-coördinate, and from s to r_1 in the x-coördinate. The loss of light due to the eclipse of $QR'Q'S$ can—apart from effects of geometrical boundary distortion—be found by integrating the same integrand between $\pm\sqrt{r_2^2 - (\delta - x)^2}$ in the y-coördinate, and from $(\delta - r_2)$ to s in the x-coördinate. If the primary star is the larger of the two and $s > r_1$, the eclipse becomes annular. The segment $QRQ'S$ then evidently vanishes, and the loss of light is obtained by integrating between $\pm\sqrt{r_2^2 - (\delta - x)^2}$ in the y-coördinate, and between $(\delta \pm r_2)$ in the x-coördinate.

Effects of distortion of the arc $QR'Q'S$ can, to the order of accuracy we are working, be finally obtained by a single integration of Δy with respect to x, taken also from $(\delta - r_2)$ to s if the eclipse is partial, and from $(\delta - r_2)$ to $(\delta + r_2)$ if it is annular. If the disk of the star undergoing eclipse is uniformly bright, the integrand consists solely of $2\Delta y$; if it is completely darkened at limb, $3\Delta y$ is to be multiplied by the foreshortening factor $\sqrt{2\delta(s - x)}$. Since Δy is a small quantity of the first order itself, the effects of gravity-darkening are clearly negligible in this connection.

Let us, following Takeda, denote terms obtained by double integration with respect to x and y within circular limits as "circular integrals," and those resulting from a single integration of Δy with respect to x as "boundary corrections." As is evident from equation (33), the latter will consist of two parts, one involving Δr_2 which depends on the distortion of the eclipsing component, and the other involving Δr_1 which depends on the distortion of the eclipsed star. Our task will be to investigate both in turn.

IV. Associated Alpha-Functions

9. In the preceding section we were led to express the changes of light due to the eclipse of a distorted star in terms of circular integrals defined as

$$\pi r_1^{m+n+2}\alpha_n^m = \left\{\int_s^{r_1}\int_{-\sqrt{r_1^2-x^2}}^{+\sqrt{r_1^2-x^2}} + \int_{\delta-r_2}^{s}\int_{-\sqrt{r_2^2-(\delta-x)^2}}^{+\sqrt{r_2^2-(\delta-x)^2}}\right\} x^m z^n dxdy \tag{34.0}$$

if the eclipse is partial, and

$$\pi r_1^{m+n+2}\alpha_n^m = \int_{\delta-r_2}^{\delta+r_2}\int_{-\sqrt{r_2^2-(\delta-x)^2}}^{+\sqrt{r_2^2-(\delta-x)^2}} x^m z^n dxdy \tag{34.1}$$

if it is annular; where

$$z = \sqrt{r_1^2 - x^2 - y^2}.$$

We propose to call the dimensionless quantities defined by the preceding equations *associated α-functions* of the order m and index n. As the reader may easily verify, the two lowest such functions of zero order possess familiar meaning: α_0^0 represents the fractional loss of light due to the eclipse of an uniformly bright circular disk (equal to the fractional area eclipsed), while $\frac{3}{2}\alpha_1^0$ gives the same loss of light if the eclipsed disk is completely darkened at limb. In the case of incomplete limb-darkening the corresponding loss of light can always be obtained as a certain weighted mean of

α_0^0 and α_1^0; but associated α-functions of higher orders and indices are required to account for effects of gravity-darkening. Thus in the present section our task will be to investigate the properties of such functions and integrate equations (34) for any value of m and n in a finite number of terms.

It is convenient, for this purpose, to split equation (34.0) in two parts by putting

$$\pi r_1^{m+n+2}\alpha_n^m = \mathfrak{A}_n^m + \mathfrak{B}_n^m, \tag{34.01}$$

where

$$\mathfrak{A}_n^m = \int_s^{r_1} \int_{-\sqrt{r_1^2-x^2}}^{+\sqrt{r_1^2-x^2}} x^m z^n \, dx\, dy, \tag{35}$$

$$\mathfrak{B}_n^m = \int_{\delta-r_2}^{s} \int_{-\sqrt{r_2^2-(\delta-x)^2}}^{+\sqrt{r_2^2-(\delta-x)^2}} x^m z^n \, dx\, dy, \tag{36}$$

and to deal with each in turn. For annular eclipses, equation (34.1) can likewise be written as

$$\pi r_1^{m+n+2}\alpha_n^m = \mathfrak{B}_n^m, \tag{34.11}$$

provided that s is replaced by $(\delta + r_2)$ in the limits of integration on the right-hand side of (36).

As to $\mathfrak{A}$, integrating we obtain at once that

$$\mathfrak{A}_n^m = B(\tfrac{1}{2}, 1 + \tfrac{1}{2}n) \int_s^{r_1} x^m (r_1^2 - x^2)^{(n+1)/2} dx \tag{37}$$

$$= B(\tfrac{1}{2}, 1 + \tfrac{1}{2}n)\{D^m{}_{n+1}(r_1) - D^m{}_{n+1}(s)\},$$

where $B(m, n)$ denotes the complete beta-functions (numerical factors), and

$$D^m{}_{2\nu}(x) = \int_0^x x^m (r_1^2 - x^2)^\nu dx,$$

which is a binomial integral tractable by elementary means.

The calculation of $\mathfrak{B}_n^m$ proves more tedious. Integrating with respect to y we find that for n zero or an even integer

$$\mathfrak{B}^m{}_{2\nu} = \frac{1}{\pi} B(\tfrac{1}{2}, 1 + \nu) \sum_{j=0}^{\nu} B(\tfrac{1}{2}, \tfrac{1}{2} + \nu - j) I^m{}_{2j,1,2(\nu-j)}; \qquad \nu = 0, 1, 2 \cdots \tag{38.0}$$

while if n is odd,

$$\mathfrak{B}^m{}_{2\nu-1} = \frac{1}{\pi} B(\tfrac{1}{2}, \tfrac{1}{2} + \nu) \left\{ \sum_{j=1}^{\nu} B(\tfrac{1}{2}, j) I^m{}_{2(\nu-j),1,2j-1} + 2\Pi^m{}_{2\nu} \right\}, \tag{38.1}$$

where

$$I^m{}_{2\alpha,\beta,\gamma} = \int_{\delta-r_2}^{p} (r_1^2 - x^2)^\alpha [r_2^2 - (\delta - x)^2]^{\beta/2} [2\delta(s - x)]^{\gamma/2} x^m dx, \tag{39}$$

and

$$\Pi^m{}_{2\nu} = \int_{\delta-r_2}^{p} x^m (r_1^2 - x^2)^\nu \sin^{-1} \sqrt{\frac{r_2^2 - (\delta - x)^2}{r_1^2 - x^2}}\, dx, \tag{40}$$

where $p = s$ or $(\delta + r_2)$ as to whether the eclipse is partial or annular. These are the two standard forms which we have to solve.

10. The methods of integration of (39) depend in principle upon the values of the three subscripts α, β, γ. The first can be easily suppressed by putting

$$I^m{}_{2\alpha,\beta,\gamma} = \sum_{j=0}^{\alpha} (-1)^j \binom{\alpha}{j} r_1^{2(\alpha-j)} I^{m+2j}_{0,\beta,\gamma}. \tag{41.0}$$

Further, the nature of our problem is such that β can assume values of odd integers only.[17] Thus the character of (39) depends on whether γ is odd or even. If it is zero or an even integer, (39) can be solved in terms of circular and algebraic functions by elementary methods.

If, however, γ is odd, (39) becomes an elliptic integral. In order to solve it we change over to a new variable

$$t = x - h, \tag{42.0}$$

where h is a constant defined so as to render

$$[r_2^2 - (\delta - x)^2][s - x] = (t - e_1)(t - e_2)(t - e_3), \tag{43}$$

subject to conditions that

$$e_1 > e_2 > e_3 \qquad \text{and} \qquad e_1 + e_2 + e_3 = 0.$$

Evidently

$$h = \tfrac{1}{3}(2\delta + s), \tag{44}$$

and

$$\left.\begin{aligned} e_1 &= +\tfrac{1}{3}(\delta - s) + r_2, \\ e_2 &= -\tfrac{2}{3}(\delta - s), \\ e_3 &= +\tfrac{1}{3}(\delta - s) - r_2, \end{aligned}\right\} \tag{45.0}$$

if the eclipse is partial, and

$$\left.\begin{aligned} e_1 &= -\tfrac{2}{3}(\delta - s), \\ e_2 &= +\tfrac{1}{3}(\delta - s) + r_2, \\ e_3 &= +\tfrac{1}{3}(\delta - s) - r_2, \end{aligned}\right\} \tag{45.1}$$

if it is annular. In either case we are therefore entitled to put

$$t = \wp(u), \tag{42.1}$$

where $\wp$ denotes the Weierstrass π-function of an argument u which replaces t as our independent variable.

The integrals on the right-hand side of (41.0) in terms of this new variable become

$$I^m{}_{0,\beta,\gamma} = -2i^{\beta+\gamma}(2\delta)^{\gamma/2} \int_{\omega_2}^{\omega_1+\omega_2} \{[\wp(u) - e_1][\wp(u) - e_3]\}^{(\beta+1)/2} \{\wp(u) - e_2\}^{(\gamma+1)/2} \{\wp(u) + h\}^m du, \tag{46.0}$$

with limits defined by

$$\begin{aligned} \wp(\omega_1) &= e_1, \\ \wp(\omega_2) &= e_3. \end{aligned} \tag{47}$$

If, as in equations (38.0) or (38.1), $\beta = 1$, then (46.0) can undergo further reduction. For, by definition, we have

$$2\sqrt{[\wp(u) - e_1][\wp(u) - e_2][\wp(u) - e_3]} = \wp'(u), \tag{48}$$

where accent denotes derivative with respect to u. Squaring (48) and inserting in (46.0) we obtain

$$I^m{}_{0,1,\gamma} = -\frac{i^{\gamma+1}}{2}(2\delta)^{\gamma/2} \int_{\omega_2}^{\omega_1+\omega_2} [\wp(u) - e_2]^{(\gamma-1)/2} [\wp(u) + h]^m [\wp'(u)]^2 du. \tag{46.1}$$

[17] Terms with β zero or an even integer do not occur in the light curve on account of symmetry.

But, if we abbreviate

$$g_2 = -4(e_1e_2 + e_1e_3 + e_2e_3), \qquad (49)$$
$$g_3 = +4e_1e_2e_3,$$

it follows from (48) that[18]

$$\{\wp'(u)\}^2 = 4\wp^3(u) - g_2\wp(u) - g_3 \qquad (50)$$

and hence, γ being odd, the whole integrand of (46.1) can be written out as a polynomial of the $\{\frac{1}{2}(\gamma - 1) + m + 3\}$th degree in powers of $\wp(u)$.

The last step in the evaluation of (46.0) or (46.1) consists in reducing integrals

$$\int_{\omega_2}^{\omega_1+\omega_2} \{\wp(u)\}^j du, \qquad j = 0, 1, 2 \cdots \qquad (A)$$

to Legendre normal forms. This can proceed by expressing, by successive differentiation of (50), the powers of $\wp(u)$ in terms of its derivatives. A general expression for $\wp^j(u)$ contains, in addition to $\wp^{\mathrm{II}(j-1)}(u)$ and lower derivatives of even orders, also the first power of $\wp(u)$ (for $j > 2$) and a constant. If we put

$$\int_{\omega_2}^{\omega_1+\omega_2} du = \omega_1 \qquad \text{and} \qquad \int_{\omega_2}^{\omega_1+\omega_2} \wp(u)du = -\eta_1$$

and remember that odd derivatives of $\wp(u)$ with arguments $\omega_1 + \omega_2$ or ω_2 vanish, we readily see [19] that

$$\int_{\omega_2}^{\omega_1+\omega_2} \{\wp(u)\}^2 du = \frac{1}{12} g_2\omega_1$$

and for $j > 2$ all integrals of powers of $\wp(u)$ can be expressed as a linear combinations of ω_1 and η_1, with coefficients involving powers and cross-products of the invariants g_2 and g_3. The functions ω_1 and η_1 rewritten in terms of Legendre normal integrals take finally the forms

$$\omega_1 = \frac{F\left(\frac{\pi}{2}, \kappa\right)}{\sqrt{e_1 - e_3}} \qquad (51)$$

and

$$\eta_1 = \sqrt{e_1 - e_3}E\left(\frac{\pi}{2}, \kappa\right) - \frac{e_1}{\sqrt{e_1 - e_3}} F\left(\frac{\pi}{2}, \kappa\right), \qquad (52)$$

where F and E denote the Legendre complete integrals of the first and second kind, with the modulus

$$\kappa^2 = \frac{e_2 - e_3}{e_1 - e_3}. \qquad (53)$$

The reader should notice that the moduli appropriate for partial and annular eclipses are mutually reciprocal.

11. After having thus established the solution of (39) in a finite number of terms for any value of the subscripts and of m, let us return to (40). Integrating II by parts we obtain

$$\Pi^m{}_{2\nu} = GD^m{}_{2\nu}(s) - \sqrt{\frac{\delta}{2}} \int_{c_3}^{c_2} \frac{XD^m{}_{2\nu}(x)dx}{\sqrt{(x - c_1)(x - c_2)(x - c_3)}}, \qquad (54)$$

[18] This is the differential equation defining $\wp(u)$; cf. Whittaker and Watson, *Modern Analysis*, Cambridge, 1920, sec. 20.22.

[19] Cf. *op. cit.*, sec. 20.52.

where $G = \frac{\pi}{2}$ or 0 as to whether the eclipse is partial or annular, X stands for

$$\frac{x^2 - 2sx + r_1^2}{r_1^2 - x^2},$$

and

$$\begin{aligned} c_1 &= \delta + r_2, \\ c_2 &= s, \\ c_3 &= \delta - r_2, \end{aligned}$$

if the eclipse is partial, and

$$\begin{aligned} c_1 &= s, \\ c_2 &= \delta + r_2, \\ c_3 &= \delta - r_2, \end{aligned}$$

if it is annular. Substitute, as before,

$$x - \frac{1}{3}\sum_{j=1}^{3} c_j = \wp(u) \tag{42.0}$$

and expand

$$XD^m{}_{2\nu}(x) = \sum_{j=0}^{m+n+2} a_n{}^m(j)\{\wp(u)\}^j + b_n{}^m r_1{}^{m+n+2}\left\{\frac{r_1 - s}{\wp(u) + h - r_1} + (-1)^m \frac{r_1 + s}{\wp(u) + h + r_1}\right\},$$

where the coefficients $a_n{}^m$ are polynomials of the $(m + n + 2 - j)$th degree in r_1, δ, and s, and $b_n{}^m$ is a positive fraction (numerical factor). Since, by definition,

$$dx = 2\sqrt{(x - c_1)(x - c_2)(x - c_3)}\, du \tag{48}$$

we see that the $\Pi^m{}_{2\nu}$ can be expressed in terms of integrals of powers of $\wp(u)$ which we have just solved, plus two integrals of the form

$$\int_{\omega_2}^{\omega_1+\omega_2} \frac{du}{\wp(u) + h \pm r_1}, \tag{B}$$

which are new and remain to be evaluated.

In order to do so we introduce new arguments $v_{1,2}$ defined by

$$-(h \pm r_1) = \wp(v_{1,2}). \tag{55.00}$$

Then, by means of a well-known theorem [20] we have

$$\int_{\omega_2}^{\omega_1+\omega_2} \frac{du}{\wp(u) - \wp(v_j)} = \frac{2}{\wp'(v_j)}\{\omega_1\zeta(v_j) - \eta_j v_j\}, \qquad j = 1, 2, \tag{56}$$

where accent denotes derivative with respect to v_j and ζ is the Weierstrass zeta-function. As one can easily verify,

$$\wp'(v_{1,2}) = \mp 2i\sqrt{2\delta}(r_1 \pm s) \tag{55.10}$$

if the eclipse is partial, and

$$\wp'(v_{1,2}) = -2i\sqrt{2\delta}(r_1 \pm s) \tag{55.11}$$

if it is annular. In order to remove the imaginary unit we put

$$\begin{aligned} v_1 &= iw_1, \\ v_2 &= iw_2 + \omega_1. \end{aligned} \tag{57}$$

Remembering that

$$\zeta(\omega_1) = \eta_1, \qquad \wp(\omega_1) = e_1, \qquad \wp'(\omega_1) = 0,$$

[20] Whittaker and Watson, *op. cit.*, sec. 20.53.

the addition-theorem for Weierstrass zeta-functions yields

$$\zeta(\omega_1 + iw_j) = \eta_1 + \zeta(iw_j) + \frac{i}{\sqrt{2\delta}}\{r_1 - (\delta - r_2)\}$$

if the eclipse is partial, and

$$\zeta(\omega_1 + iw_j) = \eta_1 + \zeta(iw_j) + i\sqrt{2\delta}$$

if it is annular. If we further substitute

$$\zeta(iw) = -i\zeta^*(w), \tag{58}$$

where

$$\zeta^*(w; e_1, e_2, e_3) = \zeta(w; -e_1, -e_2, -e_3),$$

we finally obtain that, for partial eclipses,

$$\sqrt{2\delta}(r_1 + s)\int_{\omega_2}^{\omega_1+\omega_2} \frac{du}{\wp(u) - \wp(v_1)} = \omega_1\zeta^*(w_1) + \eta_1 w_1, \tag{59.0}$$

and

$$\sqrt{2\delta}(r_1 - s)\int_{\omega_2}^{\omega_1+\omega_2} \frac{du}{\wp(u) - \wp(v_2)} = -\omega_1\zeta^*(w_2) - \eta_1 w_2 + \frac{\omega_1}{\sqrt{2\delta}}(r_1 + r_2 - \delta). \tag{59.10}$$

If the eclipse is annular, equation (59.0) continues to hold good; but (59.10) is to be replaced by

$$\sqrt{2\delta}(r_1 - s)\int_{\omega_2}^{\omega_1+\omega_2} \frac{du}{\wp(u) - \wp(v_2)} = \omega_1\zeta^*(w_2) + \eta_1 w_2 - \omega_1\sqrt{2\delta}. \tag{59.11}$$

The functions $w_{1,2}$ and $\zeta^*(w_{1,2})$, expressed in terms of Legendre normal forms, become

$$w_{1,2} = \frac{F(\phi_{1,2}, \kappa')}{\sqrt{e_1 - e_3}} \tag{60}$$

and

$$(p)\quad \zeta^*(w_{1,2}) = e_3 w_{1,2} + \sqrt{e_1 - e_3}E(\phi_{1,2}, \kappa') + \frac{1}{\sqrt{2\delta}}[r_1 \pm (\delta - r_2)], \tag{61.0}$$

$$(a)\quad \zeta^*(w_{1,2}) = e_3 w_{1,2} + \sqrt{e_1 - e_3}E(\phi_{1,2}, \kappa') + \sqrt{2\delta}, \tag{61.1}$$

—as to whether the eclipse is partial (p) or annular (a)—where κ', the complementary modulus, is defined by

$$(\kappa')^2 = \frac{e_1 - e_2}{e_1 - e_3} = 1 - \kappa^2, \tag{62}$$

and the amplitudes for partial and annular eclipses take the respective forms

$$(p)\quad \phi_1 = \sin^{-1}\sqrt{\frac{2r_2}{r_1 + r_2 + \delta}}, \qquad \phi_2 = \sin^{-1}\sqrt{\frac{2\delta}{r_1 + r_2 + \delta}}, \tag{63.0}$$

and

$$(a)\quad \phi_1 = \phi_2 = \sin^{-1}\sqrt{\frac{r_1 + r_2 - \delta}{r_1 + r_2 + \delta}}. \tag{63.1}$$

Let us put, for brevity's sake,

$$(r_1 + s)\int_{\omega_2}^{\omega_1+\omega_2} \frac{du}{\wp(u) - \wp(v_1)} \mp (r_1 - s)\int_{\omega_2}^{\omega_1+\omega_2} \frac{du}{\wp(u) - \wp(v_2)} = \frac{1}{\sqrt{2\delta}}\mathfrak{E}_{1,2}. \tag{64}$$

By combination of the above formulae it follows that, for partial eclipses,

$$\mathfrak{E}_{1,2} = \left\{ E\left(\frac{\pi}{2}, \kappa\right) - F\left(\frac{\pi}{2}, \kappa\right) \right\} \left\{ F(\phi_1, \kappa') \pm F(\phi_2, \kappa') \right\} + F\left(\frac{\pi}{2}, \kappa\right) \left\{ E(\phi_1, \kappa') \pm E(\phi_2, \kappa') + \kappa \cos \phi_1 \sec \phi_2 \right\}. \quad (65.00)$$

If the upper sign is valid, this expression admits of a drastic simplification; for, by an obvious extension of a theorem due to Legendre,[21] the reader should have no difficulty to prove that

$$\mathfrak{E}_1 = \frac{\pi}{2} + \sqrt{\frac{\delta}{r_2}}\, F\left(\frac{\pi}{2}, \kappa\right). \quad (65.10)$$

Hence, by subtraction of $\mathfrak{E}_1$ and $\mathfrak{E}_2$, the latter takes the form

$$\mathfrak{E}_2 = \frac{\pi}{2} - 2\left\{ E\left(\frac{\pi}{2}, \kappa\right) - F\left(\frac{\pi}{2}, \kappa\right) \right\} F(\phi_2, \kappa') - 2\left\{ E(\phi_2, \kappa') - \frac{1}{2}\sqrt{\frac{\delta}{r_2}} \right\} F\left(\frac{\pi}{2}, \kappa\right), \quad (65.2)$$

in which both kinds of incomplete integrals possess a common amplitude.

If, finally, the eclipse is annular we similarly obtain

$$\mathfrak{E}_1 = \kappa \sqrt{\frac{\delta}{r_2}}\, F\left(\frac{\pi}{2}, \kappa\right), \quad (65.11)$$

and

$$\mathfrak{E}_2 = 2F(\phi_{1,2}, \kappa') \left\{ E\left(\frac{\pi}{2}, \kappa\right) - F\left(\frac{\pi}{2}, \kappa\right) \right\} + 2F\left(\frac{\pi}{2}, \kappa\right) \left\{ E(\phi_{1,2}, \kappa') + \frac{\kappa}{2}\sqrt{\frac{\delta}{r_2}} \right\}. \quad (65.3)$$

12. The evaluation of circular integrals associated with effects of the tidal distortion has thus been completed. We found that the expressions for $\mathfrak{A}_n^m$ are all elementary, while those for $\mathfrak{B}_n^m$ are such only if n is zero or an even integer. If it is odd the $\mathfrak{B}$'s are found to involve elliptic integrals. Expressions of the form $I^m{}_{0,1,\gamma}$, where γ is an odd integer, or—if m is also odd—$\Pi^m{}_{2\nu}$, can all be solved in terms of Legendre complete integrals of the first and second kind. If, however, m is zero or even, the $\Pi^m{}_{2\nu}$'s involve also complete elliptic integrals of the third kind which belong to the "circular" class and are therefore expressible in terms of incomplete integrals of the first and second kind with complementary moduli.

The second-harmonic rotational distortion invokes, in addition to terms already treated, new terms involving powers of y (cf. sec. III). Terms containing odd powers of y vanish, however, on account of symmetry, while even powers of y can always be expressed in terms of those of x and z by means of the relation $x^2 + y^2 + z^2 = r_1^2$. Hence the effects of the rotational distortion (as well as the boundary corrections) can be expressed in terms of functions already treated, and the solution of our problem is thus complete.

V. Theoretical Light Curves

13. The equations governing the changes of light due to eclipses in close binary systems can now be put down at once. Let us start from

$$\Delta\mathfrak{L} = \frac{3(1-u)}{3-u}\Delta\mathfrak{L}^U + \frac{2u}{3-u}\Delta\mathfrak{L}^D, \quad (17.0)$$

where u is the coefficient of limb-darkening of the eclipsed star, and put

$$\Delta\mathfrak{L}^U = \alpha^U + f_*{}^U + f_1{}^U + f_2{}^U, \quad (66.0)$$

$$\Delta\mathfrak{L}^D = \alpha^D + f_*{}^D + f_1{}^D + f_2{}^D, \quad (66.1)$$

where α^U and α^D denote the "uniform" and "completely darkened" fractional losses of light that

[21] *Cf.* Whittaker and Watson, *op. cit.*, sec. 22.735.

would take place if the eclipsed components were spherical, and the f's represent the effects of distortion: f_* will denote effects (mainly of gravity-darkening) associated with circular integrals and expressible in terms of associated α-functions, while f_1 and f_2 stand for the boundary corrections due to the distortion of the primary (eclipsed) and secondary (eclipsing) component, respectively.

Evidently

$$\alpha^U = \alpha_0^0 \qquad \text{and} \qquad \alpha^D = \tfrac{3}{2}\alpha_1^0;$$

while by making use of equations (18.0) and (18.1), the f_*'s result as follows:

$$\begin{aligned}
f_*{}^U = {} & \frac{1}{3} v_1{}^{(2)} \left\{ \frac{1}{2} \Omega_2{}^U [3(n_0{}^2 - n_1{}^2)\alpha_2{}^0 + 3(n_2{}^2 - n_1{}^2)\alpha_0{}^2 + 6n_0 n_2 \alpha_1{}^1 + 2P_2(n_1)\alpha_0{}^0] \right. \\
& \left. + 2P_2(n_0)\alpha_0{}^0 + 3n_0 n_2 \alpha^1{}_{-1} \right\} \\
& - w_1{}^{(2)} \left\{ \frac{1}{2} \Omega_2{}^U [3l_0{}^2\alpha_2{}^0 + 6l_0 l_2 \alpha_1{}^1 + 3l_2{}^2\alpha_0{}^2 - \alpha_0{}^0] + 2P_2(l_0)\alpha_0{}^0 + 3l_0 l_2 \alpha^1{}_{-1} \right\} \\
& - w_1{}^{(3)} \left\{ \frac{1}{2} \Omega_3{}^U [5l_0{}^3\alpha_3{}^0 + 15l_0{}^2 l_2 \alpha_2{}^1 + 15l_0 l_2{}^2 \alpha_1{}^2 + 5l_2{}^3\alpha_0{}^3 - 3l_0\alpha_1{}^0 - 3l_2\alpha_0{}^1] \right. \\
& \left. + [l_0\alpha_1{}^0 + 2l_2\alpha_0{}^1]P_3'(l_0) - \frac{3}{2}\alpha_1{}^0 + \frac{3}{2} l_0 [5l_2{}^2\alpha^2{}_{-1} - l_0\alpha^0{}_{-1}] \right\} \\
& - w_1{}^{(4)} \left\{ \frac{1}{8} \Omega_4{}^U [35l_0{}^4\alpha_4{}^0 + 140l_0{}^3 l_2 \alpha_3{}^1 + 210l_0{}^2 l_2{}^2 \alpha_2{}^2 + 140l_0 l_2{}^3 \alpha_1{}^3 \right. \\
& + 35l_2{}^4\alpha_0{}^4 - 30l_0{}^2\alpha_2{}^0 - 60l_0 l_2 \alpha_1{}^1 - 30l_2{}^2\alpha_0{}^2 + 3\alpha_0{}^0] \\
& + l_0 P_4'(l_0)\alpha_2{}^0 + \frac{15}{2} l_2{}^2(7l_0{}^2 - 1)\alpha_0{}^2 - P_3'(l_0)\alpha_0{}^0 \\
& \left. + \frac{5}{2} l_0 l_2 [3(7l_0{}^2 - 2)\alpha_1{}^1 + 7l_2{}^2\alpha^3{}_{-1} - 3\alpha^1{}_{-1}] \right\} + \cdots
\end{aligned} \tag{67.0}$$

and

$$\begin{aligned}
f_*{}^D = {} & \frac{1}{2} v_1{}^{(2)} \left\{ \frac{1}{2} \Omega_2{}^D [3(n_0{}^2 - n_1{}^2)\alpha_3{}^0 + 3(n_2{}^2 - n_1{}^2)\alpha_1{}^2 + 6n_0 n_2 \alpha_2{}^1 + 2P_2(n_1)\alpha_1{}^0] \right. \\
& \left. + 4P_2(n_0)\alpha_1{}^0 + 6n_0 n_2 \alpha_0{}^1 \right\} \\
& - \frac{3}{2} w_1{}^{(2)} \left\{ \frac{1}{2} \Omega_2{}^D [3l_0{}^2\alpha_3{}^0 + 6l_0 l_2 \alpha_2{}^1 + 3l_2{}^2\alpha_1{}^2 - \alpha_1{}^0] + 4P_2(l_0)\alpha_1{}^0 + 6l_0 l_2 \alpha_0{}^1 \right\} \\
& - \frac{3}{2} w_1{}^{(3)} \left\{ \frac{1}{2} \Omega_3{}^D [5l_0{}^3\alpha_4{}^0 + 15l_0{}^2 l_2 \alpha_3{}^1 + 15l_0 l_2{}^2 \alpha_2{}^2 + 5l_2{}^3\alpha_1{}^3 - 3l_0\alpha_2{}^0 - 3l_2\alpha_1{}^1] \right. \\
& \left. + 2[l_0\alpha_2{}^0 + 2l_2\alpha_1{}^1]P_3'(l_0) + 3[5l_0 l_2{}^2\alpha_0{}^2 - l_0{}^2\alpha_0{}^0 - \alpha_2{}^0] \right\} \\
& - \frac{3}{2} w_1{}^{(4)} \left\{ \frac{1}{8} \Omega_4{}^D [35l_0{}^4\alpha_5{}^0 + 140l_0{}^3 l_2 \alpha_4{}^1 + 210l_0{}^2 l_2{}^2 \alpha_3{}^2 + 140l_0 l_2{}^3 \alpha_2{}^3 + 35l_2{}^4\alpha_1{}^4 - 30l_0{}^2\alpha_3{}^0 \right. \\
& - 60l_0 l_2 \alpha_2{}^1 - 30l_2{}^2\alpha_1{}^2 + 3\alpha_1{}^0] + 2l_0 P_4'(l_0)\alpha_3{}^0 + 15l_2{}^2(7l_0{}^2 - 1)\alpha_1{}^2 - 2P_3'(l_0)\alpha_1{}^0 \\
& \left. + 5l_0 l_2 [3(7l_0{}^2 - 2)\alpha_2{}^1 + 7l_2{}^2\alpha_0{}^3 - 3\alpha_0{}^1] \right\} + \cdots,
\end{aligned} \tag{67.1}$$

where

$$\Omega_j{}^U = \left\{ \left(\frac{2j+1}{\Delta_j} + 1 - j \right) \tau - j - 2 \right\}; \qquad \Omega_j{}^D = \Omega_j{}^U - j. \tag{68}$$

The boundary corrections f_1, arising from the distortion of the primary component, can readily be expressed in terms of integrals of the form $I^m{}_{2\alpha,\beta,\gamma}$ discussed already in section IV in connection with

the associated α-functions. If we combine equations (27), (28), and (33) and put, for brevity's sake,

$$(r_2^2 - \delta^2) I^m_{0,\beta,\gamma} + \delta I^{m+1}_{0,\beta,\gamma} = \pi r_1^q \Im^m_{\beta,\gamma}, \tag{69}$$

where

$$q = 3 + \beta + \gamma + m,$$

then

$$\begin{aligned} f_1^U = \frac{1}{3} v_1^{(2)} &\left\{ 3n_0^2 \Im^0_{-1,2} + 3n_1^2 \Im^0_{1,0} + 3n_2^2 \Im^2_{-1,0} + 6n_0 n_2 \Im^1_{-1,1} - \Im^0_{-1,0} \right\} \\ - w_1^{(2)} &\left\{ 3l_0^2 \Im^0_{-1,2} + 6l_0 l_2 \Im^1_{-1,1} + 3l_2^2 \Im^2_{-1,0} - \Im^0_{-1,0} \right\} \\ - w_1^{(3)} &\left\{ 5l_0^3 \Im^0_{-1,3} + 15 l_0^2 l_2 \Im^1_{-1,2} + 15 l_0 l_2^2 \Im^2_{-1,1} + 5 l_2^3 \Im^3_{-1,0} - 3 l_0 \Im^0_{-1,1} - 3 l_2 \Im^1_{-1,0} \right\} \\ - \frac{1}{4} w_1^{(4)} &\left\{ 35 l_0^4 \Im^0_{-1,4} + 140 l_0^3 l_2 \Im^1_{-1,3} + 210 l_0^2 l_2^2 \Im^2_{-1,2} + 140 l_0 l_2^3 \Im^3_{-1,1} + 35 l_2^4 \Im^4_{-1,0} \right. \\ &\left. - 30 l_2^2 \Im^0_{-1,2} - 60 l_0 l_2 \Im^1_{-1,1} - 30 l_2^2 \Im^2_{-1,0} + 3 \Im^0_{-1,0} \right\} + \cdots \end{aligned} \tag{70.0}$$

and

$$\begin{aligned} f_1^D = \frac{1}{2} v_1^{(2)} &\{ 3n_0^2 \Im^0_{-1,3} + 3n_1^2 \Im^0_{1,1} + 3n_2^2 \Im^2_{-1,1} + 6n_0 n_2 \Im^1_{-1,2} - \Im^0_{-1,1} \} \\ - \frac{3}{2} w_1^{(2)} &\{ 3l_0^2 \Im^0_{-1,3} + 6l_0 l_2 \Im^1_{-1,2} + 3l_2^2 \Im^2_{-1,1} - \Im^0_{-1,1} \} \\ - \frac{3}{2} w_1^{(3)} &\{ 5l_0^3 \Im^0_{-1,4} + 15 l_0^2 l_2 \Im^1_{-1,3} + 15 l_0 l_2^2 \Im^2_{-1,2} + 5 l_2^3 \Im^3_{-1,1} - 3 l_0 \Im^0_{-1,2} - 3 l_2 \Im^1_{-1,1} \} \\ - \frac{3}{8} w_1^{(4)} &\{ 35 l_0^4 \Im^0_{-1,5} + 140 l_0^3 l_2 \Im^1_{-1,4} + 210 l_0^2 l_2^2 \Im^2_{-1,3} + 140 l_0 l_2^3 \Im^3_{-1,2} + 35 l_2^4 \Im^4_{-1,1} \\ &- 30 l_0^2 \Im^0_{-1,3} - 60 l_0 l_2 \Im^1_{-1,2} - 30 l_2^2 \Im^2_{-1,1} + 3 \Im^0_{-1,1} \} + \cdots . \end{aligned} \tag{70.1}$$

The remaining part of the boundary corrections arising from the distortion of the shadow cylinder cast by the secondary (eclipsing) component follows similarly. Combining equations (30), (31), and (33) we obtain

$$\begin{aligned} (r_1/r_2)^2 f_2^U = &- \frac{1}{3} v_2^{(2)} \{ 3n_1^2 I^0_{1,0} + 3n_2^2 I^2_{-1,0} - I^0_{-1,0} \} \\ &+ w_2^{(2)} \{ 3l_2^2 I^2_{-1,0} - I^0_{-1,0} \} + w_2^{(3)} \{ 5l_2^3 I^3_{-1,0} - 3l_2 I^1_{-1,0} \} \\ &+ \frac{1}{4} w_2^{(4)} \{ 35 l_2^4 I^4_{-1,0} - 30 l_2^2 I^2_{-1,0} + 3 I^0_{-1,0} \}, \end{aligned} \tag{71.0}$$

and

$$\begin{aligned} (r_1/r_2)^2 f_2^D = &- \frac{1}{2} v_2^{(2)} \{ 3n_1^2 I^0_{1,1} + 3n_2^2 I^2_{-1,1} - I^0_{-1,1} \} \\ &+ \frac{3}{2} w_2^{(2)} \{ 3l_2^2 I^2_{-1,1} - I^0_{-1,1} \} + \frac{3}{2} w_2^{(3)} \{ 5l_2^3 I^3_{-1,1} - 3l_2 I^1_{-1,1} \} \\ &+ \frac{3}{8} w_2^{(4)} \{ 35 l_2^4 I^4_{-1,1} - 30 l_2^2 I^2_{-1,1} + 3 I^0_{-1,1} \}, \end{aligned} \tag{71.1}$$

where

$$\pi r_2^{q-2} I^m_{\beta,\gamma} = \int_{\alpha}^{\omega} [r_2^2 - (\delta - x)^2]^{\beta/2} [2\delta(s - x)]^{\gamma/2} [\delta - x]^m dx. \tag{72}$$

14. If the component undergoing eclipse happens to be the smaller of the two, then for $\delta = r_2 - r_1$ the eclipse becomes total and the loss of light equals the luminosity of the primary star. In such a case the foregoing algebra admits of drastic simplification. The boundary corrections evidently vanish, and so do the $\mathfrak{B}_n^m$'s of the circular integrals; while the normalized $\mathfrak{N}_n^m$'s degenerate for $s = -r_1$ into numerical factors. The fractional luminosity $\mathfrak{L}_1$ of the distorted primary star then takes the form

$$\begin{aligned}\mathfrak{L}_1 = 1 &+ \frac{3(1-u)}{3-u}\left\{\frac{1}{3}v_1^{(2)}\left[1+\frac{\tau}{4}\left(\frac{5}{\Delta_2}-1\right)\right]P_2(n_0) - w_1^{(2)}\left[1+\frac{\tau}{4}\left(\frac{5}{\Delta_2}-1\right)\right]P_2(l_0)\right.\\ &\left.+\frac{3}{4}w_1^{(4)}\left[1+\frac{\tau}{18}\left(\frac{9}{\Delta_4}-3\right)\right]P_4(l_0)+\cdots\right\}\\ &+\frac{2u}{3-u}\left\{\frac{8}{15}v_1^{(2)}\left[1+\frac{\tau}{4}\left(\frac{5}{\Delta_2}-1\right)\right]P_2(n_0) - \frac{8}{5}w_1^{(2)}\left[1+\frac{\tau}{4}\left(\frac{5}{\Delta_2}-1\right)\right]P_2(l_0)\right.\\ &\left.-\frac{5}{4}w_1^{(3)}\left[1+\frac{\tau}{10}\left(\frac{7}{\Delta_3}-2\right)\right]P_3(l_0)+\cdots\right\},\end{aligned} \tag{73}$$

the primary's luminosity in its undistorted state being taken as the unity of light.

Equation (73) specifies evidently the variation of light of a distorted star between eclipses. We notice that no light variation arises from the third harmonic unless there is some limb-darkening, nor any from the fourth unless the limb-darkening is incomplete. These particular properties of the even and odd harmonics higher than the second were already pointed out by Russell [22] in an early paper on the light variation of asteroids. We further notice that the effect of gravity-darkening, upon variation of total light between eclipses, arising from the second, third, and fourth tidal harmonic distortion of a centrally condensed star, is in general to multiply the variation invoked by the distorted geometry alone by the factors of 2, $\frac{3}{2}$, and $\frac{4}{3}$, respectively. These results are not wholly new. Equation similar to (73) for the special case of centrally condensed stars and total radiation has first been derived by Takeda,[23] and later generalized by Sterne [24] for stars of any structure with an arbitrary coefficient of gravity-darkening. The variation of light caused by the changing geometry alone, with coefficient of gravity-darkening appropriate for black-body radiation, were to the same order of accuracy deduced first by the writer.[25]

15. Within minima, however, the changes of light due to the eclipse of a rotationally and tidally distorted star become vastly more complex, and equations (67), (70), and (71) then represent a general and rigorous solution of the problem set forth in the introduction of the present paper. As it could have been anticipated, its form is by no means simple. In particular, effects of the second, third, and fourth spherical harmonic distortion invoke, in addition to the two fundamental modes α_0^0 and α_1^0, 22 further associated α-functions of various orders and indices summarized in the following scheme,

α^0_{-1}	α^1_{-1}	α^2_{-1}	α^3_{-1}	
	α_0^1	α_0^2	α_0^3	α_0^4
	α_1^1	α_1^2	α_1^3	α_1^4
α_2^0	α_2^1	α_2^2	α_2^3	
α_3^0	α_3^1	α_3^2		
α_4^0	α_4^1			
α_5^0				

plus an analogous number of functions of the type $\mathfrak{I}^m{}_{\beta,\gamma}$ and $I^m{}_{\beta,\gamma}$. All these functions have been defined and their properties investigated in section IV, so that the task which remains is to evaluate them explicitly in a finite number of terms.

[22] *Ap.J.*, **24**, 1, 1906.
[23] *Kyoto Mem.*, A, **17**, 197, 1934.
[24] *Proc. Nat. Acad. Sci.*, **27**, 99, 1941.
[25] *Ann. New York Acad. Sci.*, **41**, 13, 1941.

VI. Evaluation of the Integrals

16. As to the associated α-functions, in section IV we found that their character depends substantially on whether their index n is odd or even. If it is zero or an even integer, the associated α-functions assume the form

$$\pi^2 r_1^{m+2(\nu+1)}\alpha^m{}_{2\nu} = B(\tfrac{1}{2}, \nu+1)\{2G[D^m{}_{2\nu+1}(r_1) - D^m{}_{2\nu+1}(s)] + \sum_{j=0}^{\nu} B(\tfrac{1}{2}, \tfrac{1}{2}+\nu-j) I^m{}_{2j,1,2(\nu-j)}\}, \quad (74.0)$$

where $G = \pi/2$ or 0 as to whether the eclipse is partial or annular; while if the index is odd,

$$\pi^2 r_1^{m+2\nu+1}\alpha^m{}_{2\nu-1} = 2B(\tfrac{1}{2}, \tfrac{1}{2}+\nu)\left\{ GD^m{}_{2\nu}(r_1) + \frac{1}{2}\sum_{j=1}^{\nu} B(\tfrac{1}{2}, j) I^m{}_{2(\nu-j),1,2j-1} - \sqrt{\frac{\delta}{2}}\int_{c_3}^{c_2} \frac{XD^m{}_{2\nu}(x)dx}{\sqrt{(x-c_1)(x-c_2)(x-c_3)}}\right\}. \quad (74.1)$$

Functions of the former kind can be evaluated in terms of circular and algebraic functions, while those of the latter kind are expressible only by means of elliptic integrals.

Let us consider the former case first. By direct integration we easily find that, as long as the eclipse is partial,

$$\pi r_1^2\alpha_0{}^0 = r_1^2\cos^{-1}\frac{s}{r_1} + r_2^2\cos^{-1}\frac{\delta - s}{r_2} - \delta\sqrt{r_1^2 - s^2} \quad (75.0)$$

and

$$\pi r_1^3\alpha_0{}^1 = \delta r_2^2\cos^{-1}\frac{\delta - s}{r_2} - \delta(\delta - s)\sqrt{r_1^2 - s^2}; \quad (75.1)$$

while the respective functions of higher orders and indices take readily the forms:

$$\alpha_0{}^2 = \frac{1}{4}\alpha_0{}^0 + \frac{1}{4}(5D - 2S)\alpha_0{}^1 - \frac{5D}{6\pi}(1 - S^2)^{\frac{3}{2}}, \quad (75.2)$$

$$\alpha_0{}^3 = \quad + \frac{1}{4}(7D^2 - 6DS + 3)\alpha_0{}^1 - \frac{D}{6\pi}(7D + S)(1 - S^2)^{\frac{3}{2}}, \quad (75.3)$$

$$\alpha_0{}^4 = \frac{1}{8}\alpha_0{}^0 + \frac{1}{8}\{7D^2(3D - 4S) + 2(7D - 2S) + 4DS^2\}\alpha_0{}^1 - \frac{D}{60\pi}\{35D(3D - S) + 6S^2 + 49\}(1 - S^2)^{\frac{3}{2}}, \quad (75.4)$$

$$\alpha_2{}^0 = \frac{1}{2}\alpha_0{}^0 - \frac{1}{2}(3D - 2S)\alpha_0{}^1 + \frac{D}{\pi}(1 - S^2)^{\frac{3}{2}}, \quad (76.0)$$

$$\alpha_2{}^1 = \quad - 2D(D - S)\alpha_0{}^1 + \frac{4D^2}{3\pi}(1 - S^2)^{\frac{3}{2}}, \quad (76.1)$$

$$\alpha_2{}^2 = \frac{1}{12}\alpha_0{}^0 - \frac{1}{12}\{5D^2(7D - 10S) + 2(5D - S) + 8DS^2\}\alpha_0{}^1 + \frac{D}{18\pi}\{5D(7D - 3S) + 3\}(1 - S^2)^{\frac{3}{2}}, \quad (76.2)$$

$$\alpha_2{}^3 = \quad - \frac{D}{2}\{D^2(9D - 17S) + 3S(2DS - 1) + 5D\}\alpha_0{}^1 + \frac{D^2}{15\pi}\{D(9D - 8S) + 16 - S^2\}(1 - S^2)^{\frac{3}{2}}, \quad (76.3)$$

$$\alpha_4{}^0 = \frac{1}{3}\alpha_0{}^0 + \frac{1}{3}\{2D^2(5D - 8S) + D(4S^2 - 1) + 2S\}\alpha_0{}^1$$

$$- \frac{2D}{9\pi}\{2D(5D - 3S) - 3\}(1 - S^2)^{\frac{1}{2}}, \tag{77.0}$$

$$\alpha_4{}^1 = + \frac{D^2}{35}\{177(D - S)^2 + 37(1 - S^2) - 2D^2\}\alpha_0{}^1$$

$$- \frac{10D^2}{3\pi}(D - S)(1 - S^2)^{\frac{1}{2}}; \tag{77.1}$$

where we have abbreviated $\delta/r_1 = D$ and $s/r_1 = S$.

If, however, $r_1 > r_2$ and $\delta < r_1 - r_2$, the eclipse becomes annular; and the respective associated α-functions of even indices then result as follows. For $n = 0$ equation (74.0) yields

$$\pi r_1{}^{m+2}\alpha_0{}^m = 2I^m{}_{0,1,0}.$$

By direct integration we easily find that, during annular eclipse,

$$\alpha_0{}^0 = k^2, \tag{78.0}$$

$$\alpha_0{}^1 = k^2 D, \tag{78.1}$$

where k is the ratio of radii defined, as usual, by

$$k = r(\text{smaller})/r(\text{larger}).$$

Associated α-functions of zero index and higher orders then follow from a simple recursion formula

$$(m + 2)\alpha_0{}^m = (2m + 1)D\alpha_0{}^{m-1} + (m - 1)(k^2 - D^2)\alpha_0{}^{m-2} \tag{79.0}$$

and take the forms

$$\alpha_0{}^2 = k^2(D^2 + \tfrac{1}{4}k^2), \tag{78.2}$$

$$\alpha_0{}^3 = k^2(D^2 + \tfrac{3}{4}k^2)D, \tag{78.3}$$

$$\alpha_0{}^4 = k^2(D^4 + \tfrac{3}{2}k^2D^2 + \tfrac{1}{8}k^4). \tag{78.3}$$

If, finally, $n = 2$ or 4, the respective associated α-functions can be obtained by combining the preceding equations by means of the formulae

$$\alpha_2{}^m = \tfrac{2}{3}\{(1 + DS)\alpha_0{}^m - D\alpha_0{}^{m+1} - \alpha_0{}^{m+2}\} \tag{79.1}$$

and

$$\alpha_4{}^m = \tfrac{4}{5}\{\alpha_2{}^m + D^2\alpha_0{}^{m+2} + D^2S^2\alpha_0{}^m - 2D^2S\alpha_0{}^{m+1} - \tfrac{2}{3}[(1 + DS)\alpha_0{}^{m+2} + D\alpha_0{}^{m+3} + \alpha_0{}^{m+4}]\}. \tag{80}$$

17. The remaining thirteen associated α-functions of odd indices can be expressed only in terms of elliptic integrals. Let us put, for brevity's sake,

$$\pi\alpha_{2\nu-1}^{2\mu} = \frac{(-1)^\nu}{r_1{}^{2(\mu+\nu+1)-1}}\sqrt{\frac{\delta}{2}}\{\mathfrak{P}_{2\nu-1}^{2\mu}\omega_1 - \mathfrak{Q}_{2\nu-1}^{2\mu}\eta_1\} + \frac{\Gamma(\mu + \tfrac{1}{2})\Gamma(\nu + \tfrac{1}{2})}{\sqrt{\pi}\Gamma(\mu + \nu + \tfrac{3}{2})}\left\{\mathfrak{E}_2 + \frac{\pi}{2}\right\} \tag{81}$$

and

$$\pi\alpha_{2\nu-1}^{2\mu+1} = \frac{(-1)^\nu}{r_1{}^{2(\mu+\nu+1)}}\sqrt{\frac{\delta}{2}}\{\mathfrak{P}_{2\nu-1}^{2\mu+1}\omega_1 - \mathfrak{Q}_{2\nu-1}^{2\mu+1}\eta_1\}, \tag{82}$$

where ω_1, η_1, and $\mathfrak{E}_2$ are given by equations (51), (52), and (65.2) or (65.3) as to whether the eclipses are partial or annular. Then, using the procedure and abbreviations of section IV, we find that for

both partial and annular eclipses the $\mathfrak{P}$'s and $\mathfrak{Q}$'s take the following forms:

$$\mathfrak{P}^0_{-1} = 4(h - 2s), \tag{83.00}$$

$$\mathfrak{P}^1_{-1} = \frac{8}{9}(r_1^2 + 2r_2^2 - s^2), \tag{83.01}$$

$$\mathfrak{P}^2_{-1} = \frac{1}{3}\left\{\frac{2}{5}g_3 + \frac{1}{3}(2h - 2s)g_2 + 4h^2(h - 2s) + 8r_1^2(h - s)\right\}, \tag{83.02}$$

$$\mathfrak{P}^3_{-1} = \left\{\frac{5}{336}g_2^2 + \frac{1}{5}(2h - s)g_3 + \frac{1}{6}(3h^2 - 3hs + r_1^2)g_2 + (h^2 + r_1^2)(h^2 - 2hs + r_1^2)\right\}; \tag{83.03}$$

$$\mathfrak{Q}^0_{-1} = 4, \tag{83.10}$$

$$\mathfrak{Q}^1_{-1} = 4(h - s), \tag{83.11}$$

$$\mathfrak{Q}^2_{-1} = \left\{\frac{1}{5}g_2 + \frac{4}{3}h(3h - 4s) + \frac{8}{3}r_1^2\right\}, \tag{83.12}$$

$$\mathfrak{Q}^3_{-1} = \left\{\frac{1}{7}g_3 + \frac{3}{10}(2h - s)g_2 + 2(h^2 + r_1^2)(2h - s) - 4h^2s\right\}; \tag{83.13}$$

$$\mathfrak{P}_1^0 = \frac{2}{3}\left\{g_3 + \frac{1}{12}(2\delta - s)g_2 + h^2(h - 2s) + r_1^2(4s - h)\right\}, \tag{84.00}$$

$$\mathfrak{P}_1^1 = \frac{1}{2}\left\{\frac{1}{16}g_2^2 + \frac{1}{5}(8h - s)g_3 + \frac{1}{2}h(h - s)g_2 + (h^2 - r_1^2)(h^2 - 2hs + r_1^2)\right\}, \tag{84.01}$$

$$\begin{aligned}\mathfrak{P}_1^2 = \frac{1}{15}\Big\{&\frac{19}{20}g_2g_3 + \frac{5}{56}(13h - 2s)g_2^2 + \frac{1}{5}(75h^2 - 24hs + r_1^2)g_3 + \frac{1}{6}(30h^3 - 36h^2s \\ &+ 3hr_1^2 + 4r_1^2s)g_2 + 2(3h^5 - 6h^4s + h^3r_1^2 + 4h^2r_1^2s - 4hr_1^4 + 4r_1^4s)\Big\},\end{aligned} \tag{84.02}$$

$$\begin{aligned}\mathfrak{P}_1^3 = \frac{1}{12}\Big\{&\frac{5}{112}g_2^3 + \frac{2}{5}g_3^2 + \frac{1}{15}(39h - 4s)g_2g_3 + \frac{1}{168}(294h^2 - 100hs + 5r_1^2)g_2^2 \\ &+ \frac{2}{5}(38h^3 - 20h^2s + 2hr_1^2 + r_1^2s)g_3 + \frac{1}{3}(15h^4 - 20h^3s + 3h^2r_1^2 + 3hr_1^2s - r_1^4)g_2 \\ &+ 2(2h^6 - 4h^5s + h^4r_1^2 + 2h^3r_1^2s - 2h^2r_1^4 + 2hr_1^4s - r_1^6)\Big\},\end{aligned} \tag{84.03}$$

$$\begin{aligned}\mathfrak{P}_1^4 = \frac{1}{35}\Big\{&\frac{67}{168}g_2^2g_3 + \frac{25}{7392}(175h - 18s)g_2^3 + \frac{4}{11}(14h - 2s)g_3^2 \\ &+ \frac{1}{10}(175h^2 - 40hs + 2r_1^2)g_2g_3 + \frac{5}{168}(287h^3 - 150h^2s + 15hr_1^2 + 4r_1^2s)g_2^2 \\ &+ \frac{2}{5}(140h^4 - 100h^3s + 15h^2r_1^2 + 8hr_1^2s - 4r_1^4)g_3 \\ &+ \frac{1}{6}(105h^5 - 150h^4s + 30h^3r_1^2 + 24h^2r_1^2s - 12hr_1^4 + 4r_1^4s)g_2 \\ &+ 2(5h^7 - 10h^6s + 3h^5r_1^2 + 4h^4r_1^2s - 4h^3r_1^4 + 4h^2r_1^4s - 4hr_1^6 + 4r_1^6s)\Big\};\end{aligned} \tag{84.04}$$

$$\mathfrak{Q}_1^0 = \frac{4}{9}(7r_2^2 - 4r_1^2 + \delta^2), \tag{84.10}$$

$$\mathfrak{Q}_1^1 = \frac{1}{2}\left\{ g_3 + \frac{1}{10}(14h - 3s)g_2 + 2(2h^3 - 3h^2s + r_1^2 s) \right\}, \tag{84.11}$$

$$\mathfrak{Q}_1^2 = \frac{1}{15}\left\{ \frac{27}{40} g_2^2 + \frac{12}{7}(10h - s)g_3 + \frac{3}{10}(50h^2 - 24hs + r_1^2)g_2 + 2(15h^4 - 24h^3 s + 3h^2 r_1^2 + 8hr_1^2 s - 4r_1^4) \right\}, \tag{84.12}$$

$$\mathfrak{Q}_1^3 = \frac{1}{12}\left\{ \frac{36}{35} g_2 g_3 + \frac{1}{30}(57h - 7s)g_2^2 + \frac{2}{7}(84h^2 - 20hs + r_1^2)g_3 + \frac{3}{5}(28h^3 - 20h^2 s + 2hr_1^2 + r_1^2 s)g_2 + 4(6h^5 - 10h^4 s + 2h^3 r_1^2 + 3h^2 r_1^2 s - 2hr_1^4 + r_1^4 s) \right\}, \tag{84.13}$$

$$\mathfrak{Q}_1^4 = \frac{1}{35}\left\{ \frac{7}{32} g_2^3 + \frac{10}{7} g_3^2 + \frac{1}{154}(2051h - 174s)g_2 g_3 + \frac{7}{40}(75h^2 - 20hs + r_1^2)g_2^2 + \frac{2}{7}(385h^3 - 150h^2 s + 15hr_1^2 + 4r_1^2 s)g_3 + \frac{1}{10}(665h^4 - 600h^3 s + 90h^2 r_1^2 + 90h^2 r_1^2 + 48hr_1^2 s - 12r_1^4)g_2 + 2(35h^6 - 60h^5 s + 15h^4 r_1^2 + 16h^3 r_1^2 s - 12h^2 r_1^4 + 8hr_1^4 s - 4r_1^6) \right\}; \tag{84.14}$$

$$\mathfrak{P}_3^0 = \frac{1}{20}\left\{ \frac{19}{20} g_2 g_3 + \frac{5}{24}(7\delta + s)g_2^2 + \frac{1}{5}(75h^2 - 24hs - 60\delta e_2 - 49r_1^2)g_3 + \frac{1}{3}(15h^3 - 18h^2 s - 6hr_1^2 + 7r_1^2 s)g_2 + 2(3h^5 - 6h^4 s - 4h^3 r_1^2 + 14h^2 r_1^2 s + hr_1^4 - 16r_1^4 s) \right\}, \tag{85.00}$$

$$\mathfrak{P}_3^1 = \frac{1}{16}\left\{ \frac{5}{112} g_2^3 + \frac{2}{5} g_3^2 + \frac{1}{15}(38\delta + 9s)g_2 g_3 + \frac{1}{84}[147h^2 - 50hs - 29r_1^2 - 32\delta(e_2 - h)]g_2^2 + \frac{4}{5}[19h^3 - 10h^2 s - h(11r_1^2 + 12\delta e_2) + 2r_1^2 s]g_3 + \frac{1}{3}(15h^4 - 20h^3 s - 6h^2 r_1^2 + 12hr_1^2 s - r_1^4)g_2 + 4(h^5 - 2h^5 s - h^4 r_1^2 + 4h^3 r_1^2 s - h^2 r_1^4 - 2hr_1^4 s + r_1^6) \right\}, \tag{85.01}$$

$$\mathfrak{P}_3^2 = \frac{1}{140}\left\{ \frac{67}{56} g_2^2 g_3 + \frac{25}{672}(42\delta + 11s)g_2^3 + 2(7\delta + 2s)g_3^2 + \frac{1}{20}(1050h^2 - 240hs + 280\delta^2 - 121r_1^2)g_2 g_3 + \frac{5}{168}[861h^3 + 2(56\delta - 225s)h^2 - 4(57r_1^2 + 56\delta e_2)h + 54r_1^2 s]g_2^2 + \frac{1}{5}[840h^4 - 600h^3 s - 5(63r_1^2 + 24s^2 + 84\delta e_2)h^2 + 216hr_1^2 s - 19r_1^4]g_3 + \frac{1}{6}[315h^5 - 450h^4 s - 120h^3 r_1^2 + 324h^2 r_1^2 s - 57hr_1^4 - 16r_1^4 s]g_2 + 2[15h^7 - 30h^6 s - 12h^5 r_1^2 + 54h^4 r_1^2 s - 19h^3 r_1^4 - 16h^2 r_1^4 s + 16hr_1^6 - 16r_1^6 s] \right\}; \tag{85.02}$$

$$\mathfrak{Q}_2^0 = \frac{1}{20}\left\{\frac{27}{40}g_2^2 + 4(5\delta + s)g_3 + \frac{1}{5}(75h^2 - 36hs - 36r_1^2 - 40\delta e_2)g_2 + 2(15h^4 - 24h^3s - 12h^2r_1^2 + 28hr_1^2s + r_1^4)\right\}, \tag{85.10}$$

$$\mathfrak{Q}_2^1 = \frac{1}{16}\left\{\frac{36}{35}g_2g_3 + \frac{1}{5}(9\delta + 2s)g_2^2 + \frac{8}{7}[21h^2 - 5hs - 5r_1^2 - 6\delta(e_2 - h)]g_3 + \frac{4}{5}[21h^3 - 15h^2s - h(9r_1^2 + 8\delta e_2) + 3r_1^2s]g_2 + 8[3h^5 - 5h^4s - 2h^3r_1^2 + 6h^2r_1^2s - hr_1^4 - r_1^4s]\right\}, \tag{85.11}$$

$$\mathfrak{Q}_2^2 = \frac{1}{70}\left\{\frac{273}{832}g_2^3 + \frac{15}{7}g_3^2 + \frac{1}{28}(504\delta + 139s)g_2g_3 + \frac{7}{240}(675h^2 - 180hs + 160\delta^2 - 72r_1^2)g_2^2 + \frac{3}{7}[385h^3 + 10(7\delta - 15s)h^2 - 5(25r_1^2 + 28\delta e_2)h + 18r_1^2s]g_3 + \frac{1}{20}[1995h^4 - 1800h^3s - 20(21r_1^2 + 18s^2 + 28\delta e_2)h^2 + 648hr_1^2s - 57r_1^4]g_2 + [105h^6 - 180h^5s - 60h^4r_1^2 + 216h^3r_1^2s - 57h^2r_1^4 - 32hr_1^4s + 16r_1^6]\right\}; \tag{85.12}$$

$$\mathfrak{P}_5^0 = \frac{1}{56}\left\{\frac{67}{168}g_2^2g_3 + \frac{25}{2016}(42\delta + 11s)g_2^3 + \frac{2}{3}(7\delta + 2s)g_3^2 + \frac{1}{30}[240h(2h + \delta) + 252\delta^2 - 127r_1^2]g_2g_3 + \frac{1}{504}[20\delta(253h^2 - 76r_1^2) - 15h(163h^2 + 71r_1^2) - 224\delta e_2(5h + 8\delta)]g_2^2 + \frac{1}{5}[385h^4 - 320h^2r_1^2 + 108r_1^4 - 140\delta e_2(h^2 - r_1^2) - 8hs(25h^2 - 16r_1^2) + 224\delta^2e_2^2]g_3 + \frac{1}{12}[105h^5 - 150h^4s - 110h^3r_1^2 + 192h^2r_1^2s + 9hr_1^4 - 38r_1^4s]g_2 + [5h^7 - 10h^6s - 11h^5r_1^2 + 32h^4r_1^2s + 3h^3r_1^4 - 38h^2r_1^4s + 3hr_1^6 + 32r_1^6s]\right\}; \tag{86.00}$$

$$\mathfrak{Q}_5^0 = \frac{1}{28}\left\{\frac{7}{64}g_2^3 + \frac{5}{7}g_3^2 + \frac{1}{84}(504\delta + 139s)g_2g_3 + \frac{7}{720}[45h(11h + 8\delta) + 288\delta^2 - 153r_1^2]g_2^2 + \frac{1}{7}[2\delta(185h^2 - 67r_1^2) - 13h(5h^2 + 13r_1^2) - 28\delta e_2(5h + 8\delta)]g_3 + \frac{1}{60}[15h^2(161h^2 - 122r_1^2) - 72hs(25h^2 - 16r_1^2) + 447r_1^4 - 560\delta e_2(h^2 - r_1^2) + 896\delta^2e_2^2]g_2 + [35h^6 - 60h^5s - 55h^4r_1^2 + 128h^3r_1^2s + 9h^2r_1^4 - 76hr_1^4s + 3r_1^6]\right\}. \tag{86.10}$$

18. A synopsis of the associated α-functions required for the solution of our problem is now complete. As is evident from the preceding equations, all these functions can be made to depend on two independent parameters, and tabulated in terms of such parameters similarly as has been done for α^U and α^D. If, as usual, the ratio of the radii of both components k and the geometrical depth of the eclipse p are adopted as independent variables and defined by

$$k = \frac{r_s}{r_l} \quad \text{and} \quad p = \frac{\delta - r_l}{r_s},$$

where r_l and r_s are radii of the larger and the smaller component respectively, two different sets of tables of the $\alpha_n^m(k, p)$'s will have to be computed, depending on whether the eclipsed component is the larger or the smaller of the two.

As to the particular values which the associated α-functions may assume,

$$\alpha_n^m(k, 1) = 0 \tag{87.0}$$

for every value of m or n. All associated α-functions must vanish at the beginning of the eclipse. If $r_1 < r_2$ and m is odd, the respective associated α-functions vanish also at the moment of the inner contact and hence

$$\alpha_n^{2\mu+1}(k, \pm 1) = 0; \tag{87.1}$$

while if m is zero or an even integer,

$$\alpha_{2\nu}^{2\mu}(k, -1) = \frac{\nu!\Gamma(\mu + \frac{1}{2})}{\sqrt{\pi}(\mu + \nu + 1)!}, \tag{88.0}$$

$$\alpha_{2\nu-1}^{2\mu}(k, -1) = \frac{\Gamma(\mu + \frac{1}{2})\Gamma(\nu + \frac{1}{2})}{\sqrt{\pi}\Gamma(\mu + \nu + \frac{3}{2})}, \tag{88.1}$$

as to whether the index n is odd or even.

If, on the other hand, $r_1 > r_2$, the $\alpha_n^m(k, p)$'s at the moment of internal tangency ($p = -1$) remain functions of k. In particular, the three such functions of lowest orders and indices take the forms

$$\alpha_0^0(k, -1) = k^2, \tag{89.0}$$

$$\alpha_0^1(k, -1) = k^2(1 - k), \tag{89.1}$$

$$\alpha_1^0(k, -1) = \frac{4}{3\pi}\{\sin^{-1}\sqrt{k} + \tfrac{1}{3}(4k - 3)(2k + 1)\sqrt{k(1 - k)}\}. \tag{89.2}$$

If, finally, $r_1 > r_2$ and the eclipse is central ($\delta = 0$), a glance at equations (67.0) or (67.1) reveals that coefficients of all associated α-functions of superscripts other than zero vanish; while the remaining functions of zero order reduce to

$$\alpha_n^0(k, -1/k) = \frac{2}{n + 2}\{1 - (1 - k^2)^{1+\frac{1}{2}n}\}. \tag{90}$$

Numerically, all associated α-functions are quantities of zero order the values of which diminish in general with increasing m and n so that

$$\alpha_n^{m+1} \leqslant \alpha^m{}_{n+1}.$$

19. The boundary corrections f_1, due to the distortion of the primary component, were found (cf. equation 69) to be expressible in terms of integrals of the form $I^m{}_{0,\beta,\gamma}$ given by equation (39). In our present case $\beta = -1$ throughout—except for a single rotational term with the coefficient n_1^2 where $\beta = +1$. To the order of accuracy we are working, twenty-six integrals of the form $I^m{}_{0,-1,\gamma}$ are required to express the boundary corrections due to the tidal distortion of the eclipsed component, while four additional integrals $I^m{}_{0,1,\gamma}$ are needed to account for effects of the centrifugal force.

The solution of the respective integrals has been investigated in section IV. Those with γ zero or an even integer can again be expressed in terms of circular and algebraic functions, while if γ is odd, the integrals are elliptical. As to the former kind, by direct integration we easily find that, if the eclipse is partial,

$$I^0{}_{0,-1,0} = \cos^{-1}\frac{\delta - s}{r_2}, \tag{91.0}$$

$$I^1{}_{0,-1,0} = \delta\cos^{-1}\frac{\delta - s}{r_2} - \sqrt{r_1^2 - s^2}, \tag{91.1}$$

and with the aid of the recursion formula

$$mI^m{}_{0,-1,0} = (2m - 1)\delta I^{m-1}_{0,-1,0} + (m - 1)(r_2^2 - \delta^2)I^{m-2}_{0,-1,0} - s^{m-1}\sqrt{r_1^2 - s^2}, \tag{92}$$

it follows that

$$I^2_{0,-1,0} = \frac{1}{2}(2\delta^2 + r_2^2)I^0_{0,-1,0} - \frac{1}{2}(3\delta + s)\sqrt{r_1^2 - s^2}, \tag{91.2}$$

$$I^3_{0,-1,0} = \frac{\delta}{2}(2\delta^2 + 3r_2^2)I^0_{0,-1,0} - \frac{1}{6}(11\delta^2 + 5\delta s + 2s^2 + 4r_2^2)\sqrt{r_1^2 - s^2}, \tag{91.3}$$

$$I^4_{0,-1,0} = \frac{1}{8}(8\delta^4 + 24\delta^2 r_2^2 + 3r_2^4)I^0_{0,-1,0} - \frac{1}{24}\{50\delta^3 + 26\delta^2 s + (55r_2^2 + 14s^2)\delta + 6s^3 + 9r_2^2 s\}\sqrt{r_1^2 - s^2}, \tag{91.4}$$

$$I^5_{0,-1,0} = \frac{\delta}{8}(8\delta^4 + 40\delta^2 r_2^2 + 15r_2^4)I^0_{0,-1,0} - \frac{1}{120}\{274\delta^4 + 154\delta^3 s + (607r_2^2 + 94s^2)\delta^2 + (161r_2^2 + 54s^2)\delta s + 8(4r_2^2 + 3s^2)s^2 + 64r_2^4\}\sqrt{r_1^2 - s^2}. \tag{91.5}$$

Expressions for $I^m_{0,-1,2}(0 \leqslant m \leqslant 4)$ and $I^m_{0,-1,4}(0 \leqslant m \leqslant 2)$ can be obtained as a linear combination of the preceding formulae by means of the equations

$$I^m_{0,-1,2} = 2\delta s I^m_{0,-1,0} - 2\delta I^{m+1}_{0,-1,0} \tag{93}$$

and

$$I^m_{0,-1,4} = 4\delta^2 s^2 I^m_{0,-1,0} - 8\delta^2 s I^{m+1}_{0,-1,0} + 4\delta^2 I^{m+2}_{0,-1,0}, \tag{94}$$

which are simple and make it unnecessary to write down the eight respective expressions in full. The two elementary rotational terms take finally the forms

$$I^0_{0,1,0} = \frac{1}{2}r_2^2 \cos^{-1}\frac{\delta - s}{r_2} - \frac{1}{2}(\delta - s)\sqrt{r_1^2 - s^2}, \tag{95.0}$$

$$I^1_{0,1,0} = \delta I^0_{0,1,0} - \frac{1}{3}(r_1^2 - s^2)^{\frac{3}{2}}. \tag{95.1}$$

If the eclipse is annular, the foregoing formulae simplify considerably. In particular, if $\gamma = 0$,

$$I^0_{0,-1,0} = \pi, \tag{96.0}$$

$$I^1_{0,-1,0} = \pi\delta, \tag{96.1}$$

and, by means of the recursion formula

$$mI^m_{0,-1,0} = (2m - 1)\delta I^{m-1}_{0,-1,0} + (m - 1)(r_2^2 - \delta^2)I^{m-2}_{0,-1,0} \tag{97}$$

it follows that

$$I^2_{0,-1,0} = \pi\left(\delta^2 + \frac{1}{2}r_2^2\right), \tag{96.2}$$

$$I^3_{0,-1,0} = \pi\left(\delta^2 + \frac{3}{2}r_2^2\right)\delta, \tag{96.3}$$

$$I^4_{0,-1,0} = \pi\left(\delta^4 + 3\delta^2 r_2^2 + \frac{3}{8}r_2^4\right), \tag{96.4}$$

$$I^5_{0,-1,0} = \pi\left(\delta^4 + 5\delta^2 r_2^2 + \frac{15}{8}r_2^4\right)\delta. \tag{96.5}$$

Expressions of the form $I^m_{0,-1,2}$ (for $0 \leqslant m \leqslant 4$) and $I^m_{0,-1,4}$ (for $0 \leqslant m \leqslant 2$) can be obtained as linear combination of the preceding formulae by means of equations (93) and (94) which continue to hold good.

The two elementary rotational terms for annular eclipses become

$$I^0_{0,1,0} = \frac{1}{2}\pi r_2^2 \tag{98.0}$$

and

$$I^1{}_{0,1,0} = \frac{1}{2}\pi\delta r_2{}^2. \tag{98.1}$$

20. If γ is odd, however, equation (46.0) yields

$$I^m{}_{0,-1,\gamma} = -2i^{\gamma-1}(2\delta)^{\gamma/2}\int_{\omega_2}^{\omega_1+\omega_2}\{\wp(u)-e_2\}^{(\gamma+1)/2}\{\wp(u)+h\}^m du. \tag{46.2}$$

The solution of this integral can proceed along the line investigated in section IV; there are no practicable recursion formulae to shorten the procedure and the respective expressions have to be put down in full. If $\gamma = 1$, then

$$I^0{}_{0,-1,1} = -\frac{4}{3}\sqrt{\frac{\delta}{2}}\{2(\delta-s)\omega_1 - 3\eta_1\}, \tag{99.0}$$

$$I^1{}_{0,-1,1} = -\frac{4}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{1}{4}g_2 + 2h(\delta-s)\right\}\omega_1 + \frac{4}{3}\sqrt{\frac{\delta}{2}}\{4\delta - s\}\eta_1, \tag{99.1}$$

$$I^2{}_{0,-1,1} = -\frac{4}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{3}{10}g_3 + \frac{1}{2}\delta g_2 + 2h^2(\delta-s)\right\}\omega_1 + \frac{4}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{9}{20}g_2 + 3h(2\delta-s)\right\}\eta_1, \tag{99.2}$$

$$\begin{aligned} I^3{}_{0,-1,1} = &-\frac{4}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{5}{112}g_2{}^2 + \frac{1}{10}(8\delta+s)g_3 + \frac{1}{4}h(4\delta-s)g_2 + 2h^3(\delta-s)\right\}\omega_1 \\ &+\frac{4}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{3}{7}g_3 + \frac{3}{20}(8\delta+s)g_2 + h^2(8\delta-5s)\right\}\eta_1, \end{aligned} \tag{99.3}$$

$$\begin{aligned} I^4{}_{0,-1,1} = &-\frac{4}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{1}{10}g_2g_3 + \frac{5}{168}(5\delta+s)g_2{}^2 + \frac{1}{5}h(10\delta-s)g_3\right. \\ &\left.+\frac{1}{3}h^2(5\delta-2s)g_2 + 2h^4(\delta-s)\right\}\omega_1 \\ &+\frac{4}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{7}{80}g_2{}^2 + \frac{2}{7}(5\delta+s)g_3 + \frac{3}{10}h(10\delta-s)g_2 + h^3(10\delta-7s)\right\}\eta_1, \end{aligned} \tag{99.4}$$

$$\begin{aligned} I^5{}_{0,-1,1} = &-\frac{4}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{45}{4928}g_2{}^3 + \frac{3}{55}g_3{}^2 + \frac{1}{10}(4\delta+s)g_2g_3 + \frac{25}{56}\delta h g_2{}^2 + h^2(4\delta-s)g_3\right. \\ &\left.+\frac{5}{4}h^3(2\delta-s)g_2 + 2h^5(\delta-s)\right\}\omega_1 \\ &+\frac{4}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{261}{1540}g_2g_3 + \frac{7}{80}(4\delta+s)g_2{}^2 + \frac{30}{7}\delta h g_3 + \frac{3}{2}h^2(4\delta-s)g_2 + 3h^4(4\delta-3s)\right\}\eta_1. \end{aligned} \tag{99.5}$$

If $\gamma = 3$,

$$I^0{}_{0,-1,3} = \frac{8}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{1}{4}g_2 + \frac{4}{3}(\delta-s)^2\right\}\delta\omega_1 - \frac{32}{3}\sqrt{\frac{\delta}{2}}(\delta-s)\delta\eta_1, \tag{100.0}$$

$$\begin{aligned} I^1{}_{0,-1,3} = &\frac{8}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{3}{10}g_3 + \frac{1}{4}(2\delta-s)g_2 + \frac{4}{3}h(\delta-s)^2\right\}\delta\omega_1 \\ &-\frac{8}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{9}{20}g_2 + 4\delta(\delta-s)\right\}\delta\eta_1, \end{aligned} \tag{100.1}$$

$$I^2_{0,-1,3} = \frac{8}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{5}{112}g_2^2 + \frac{1}{5}(4\delta - s)g_3 + \frac{1}{12}(8\delta^2 - 4\delta s - s^2)g_2 + \frac{4}{3}h^2(\delta - s)^2\right\}\delta\omega_1$$
$$- \frac{8}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{7}{3}g_3 + \frac{3}{10}(4\delta - s)g_2 + \frac{4}{3}h(\delta - s)(4\delta - s)\right\}\delta\eta_1, \tag{100.2}$$

$$I^3_{0,-1,3} = \frac{8}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{1}{10}g_2g_3 + \frac{5}{336}(10\delta - s)g_2^2 + \frac{1}{30}(40\delta^2 - 8\delta s - 5s^2)g_3\right.$$
$$\left. + \frac{1}{36}h(40\delta^2 - 32\delta s + s^2)g_2 + \frac{4}{3}(\delta - s)^2h^3\right\}\delta\omega_1$$
$$- \frac{8}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{7}{80}g_2^2 + \frac{1}{7}(10\delta - s)g_3 + \frac{1}{20}(40\delta^2 - 8\delta s - 5s^2)g_2\right.$$
$$\left. + \frac{4}{3}h^2(\delta - s)(5\delta - 2s)\right\}\delta\eta_1. \tag{100.3}$$

If, finally, $\gamma = 5$,

$$I^0_{0,-1,5} = -\frac{16}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{3}{10}g_3 + \frac{1}{2}(\delta - s)g_2 + \frac{8}{9}(\delta - s)^3\right\}\delta^2\omega_1$$
$$+ \frac{16}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{9}{20}g_2 + 4(\delta - s)^2\right\}\delta^2\eta_1, \tag{101.0}$$

$$I^1_{0,-1,5} = -\frac{16}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{5}{112}g_2^2 + \frac{1}{10}(8\delta - 5s)g_3 + \frac{1}{6}(\delta - s)(4\delta - s)g_2 + \frac{8}{9}h(\delta - s)^3\right\}\delta^2\omega_1$$
$$+ \frac{16}{3}\sqrt{\frac{\delta}{2}}\left\{\frac{3}{7}g_3 + \frac{3}{20}(8\delta - 5s)g_2 + \frac{4}{9}(\delta - s)^2(8\delta + s)\right\}\delta^2\eta_1. \tag{101.1}$$

The two remaining rotational terms are of the form (46.1) and become

$$I^0_{0,1,1} = \sqrt{\frac{\delta}{2}}\left\{\frac{2}{5}g_2\eta_1 - \frac{3}{5}g_3\omega_1\right\}, \tag{102.0}$$

$$I^1_{0,1,1} = \sqrt{\frac{\delta}{2}}\left\{\frac{3}{7}g_3 + \frac{2}{5}hg_2\right\}\eta_1 - \sqrt{\frac{\delta}{2}}\left\{\frac{1}{42}g_2^2 + \frac{3}{5}hg_3\right\}\omega_1. \tag{102.1}$$

Equations (99)–(102) are valid irrespective of whether the eclipse is partial or annular.

21. The remaining part f_2 of the boundary corrections, due to the distortion of the eclipsing component, can be expressed in terms of integrals of the form $I^m_{\beta,\gamma}$ as defined by equation (72). These integrals are evidently of the same form as the previously treated $I^m_{\alpha,\beta,\gamma}$'s, and the methods of solution investigated in section IV are therefore applicable. Equations (71.0) and (71.1) show that, to the order of accuracy we are working, terms associated with tidal distortion are of the form $I^m_{-1,0}$ if the eclipsed star is uniformly bright, and $I^m_{-1,1}$ in the case of complete darkening. The former terms are all elementary and can be dealt with in terms of circular and algebraic functions; the latter are found to involve elliptic integrals.

As to terms of the former kind we easily establish by direct integration that, if the eclipse is partial,

$$\pi I^0_{-1,0} = \cos^{-1}\xi, \tag{103.0}$$

$$\pi I^1_{-1,0} = \sqrt{1 - \xi^2}, \tag{103.1}$$

where

$$\xi = \frac{\delta - s}{r_2},$$

and with the aid of the recursion formula

$$\pi m I^{m}_{-1,0} = \pi(m-1) I^{m-2}_{-1,0} + \xi^{m-1}\sqrt{1-\xi^2}, \tag{104}$$

it follows that

$$I^{2}_{-1,0} = \frac{1}{2\pi}\{\cos^{-1}\xi + \xi\sqrt{1-\xi^2}\}, \tag{103.2}$$

$$I^{3}_{-1,0} = \frac{1}{3\pi}\{\xi^2+2\}\sqrt{1-\xi^2}, \tag{103.3}$$

$$I^{4}_{-1,0} = \frac{1}{8\pi}\{3\cos^{-1}\xi + \xi(2\xi^2+3)\sqrt{1-\xi^2}\}. \tag{103.4}$$

During annular phase these expressions reduce to

$$I^{2\mu}_{-1,0} = \binom{\mu - \frac{1}{2}}{\mu} \quad \text{and} \quad I^{2\mu+1}_{-1,0} = 0, \tag{105}$$

as to whether their superscript is even or odd.

In the case of complete darkening at limb the integrals associated with tidal distortion are of the form $I^{m}_{-1,1}$ which take the explicit forms

$$\pi r_2 I^{0}_{-1,1} = 2\sqrt{2\delta}\{\eta_1 + e_2\omega_1\}, \tag{106.0}$$

$$\pi r_2^2 I^{1}_{-1,1} = \sqrt{2\delta}\left\{ e_2\eta_1 + \left(\frac{1}{6}g_2 - e_2^2\right)\omega_1 \right\}, \tag{106.1}$$

$$\pi r_2^3 I^{2}_{-1,1} = \frac{1}{10}\sqrt{2\delta}\{3g_2 - 15e_2^2\}\eta_1 - \frac{1}{10}\sqrt{2\delta}\{2g_3 - 5e_2^3\}\omega_1, \tag{106.2}$$

$$\pi r_2^4 I^{3}_{-1,1} = \frac{1}{20}\sqrt{2\delta}\left\{ 13e_2^3 - \frac{19}{7}g_3 \right\}\eta_1 + \frac{1}{20}\sqrt{2\delta}\left\{ \frac{25}{42}g_2^2 + \frac{9}{2}e_2g_3 - 15e_2^4 \right\}\omega_1, \tag{106.3}$$

$$\pi r_2^5 I^{4}_{-1,1} = \frac{1}{120}\sqrt{2\delta}\left\{ 7g_2^2 + \frac{366}{7}e_2g_3 - 177e_2^4 \right\}\eta_1 - \frac{1}{120}\sqrt{2\delta}\left\{ 8g_2g_3 + \frac{25}{7}e_2g_2^2 + 8e_2^2g_3 - 95e_2^5 \right\}\omega_1. \tag{106.4}$$

These equations are again valid irrespective of whether the eclipse is partial or annular. The $I^{m}_{-1,1}$'s can easily be rewritten in terms of complete elliptic integrals of the first and second kind; doing so we obtain

$$I^{0}_{-1,1} = \frac{2}{\pi}\sqrt{\frac{\delta}{r_2}}\{2E - (1+\xi)F\}, \tag{106.01}$$

$$I^{1}_{-1,1} = \frac{2}{3\pi}\sqrt{\frac{\delta}{r_2}}\{(1+\xi)F - 2\xi E\}, \tag{106.11}$$

$$I^{2}_{-1,1} = \frac{2}{15\pi}\sqrt{\frac{\delta}{r_2}}\{2(9-2\xi^2)E - (1+\xi)(9-2\xi)F\}, \tag{106.21}$$

$$I^{3}_{-1,1} = \frac{2}{105\pi}\sqrt{\frac{\delta}{r_2}}\{(1+\xi)(25-6\xi+8\xi^2)\}F - \frac{4}{105\pi}\sqrt{\frac{\delta}{r_2}}\{19\xi+8\xi^3\}E, \tag{106.31}$$

$$I^{4}_{-1,1} = \frac{4}{315\pi}\sqrt{\frac{\delta}{r_2}}\{147 - 24\xi^2 - 16\xi^4\}E - \frac{2}{315\pi}\sqrt{\frac{\delta}{r_2}}\{(1+\xi)(147-36\xi+12\xi^2-16\xi^3)\}F, \tag{106.41}$$

if the eclipse is partial, and

$$I^0_{-1,1} = \frac{4}{\pi\kappa}\sqrt{\frac{\delta}{r_2}}\,E, \tag{106.02}$$

$$I^1_{-1,1} = \frac{4}{3\pi\kappa}\sqrt{\frac{\delta}{r_2}}\,\{(1+\xi)F - \xi E\}, \tag{106.12}$$

$$I^2_{-1,1} = \frac{4}{15\pi\kappa}\sqrt{\frac{\delta}{r_2}}\,\{(9-2\xi^2)E + 2\xi(1+\xi)F\}, \tag{106.22}$$

$$I^3_{-1,1} = \frac{4}{105\pi\kappa}\sqrt{\frac{\delta}{r_2}}\,\{(1+\xi)(25+8\xi^2)F - \xi(19+8\xi^2)E\}, \tag{106.32}$$

$$I^4_{-1,1} = \frac{4}{315\pi\kappa}\sqrt{\frac{\delta}{r_2}}\,\{(147-24\xi^2-16\xi^4)E + 4\xi(1+\xi)(9+4\xi^2)F\}, \tag{106.42}$$

if it is annular.

The rotational distortion of the eclipsing component invokes, in addition to terms already treated, new terms of the form $I^0_{1,0}$, if the eclipsed star is uniformly bright, and $I^0_{1,1}$ in the case of complete darkening. The former integral is an elementary one; for we obtain at once that

$$I^0_{1,0} = I^0_{-1,0} - I^2_{-1,0}. \tag{107}$$

$I^0_{1,1}$ is an elliptic integral, the evaluation of which yields

$$\pi r_2^3 I^0_{1,1} = \frac{1}{5}\sqrt{\frac{\delta}{2}}\,\{2g_2\eta_1 - 3g_3\omega_1\} \tag{108.0}$$

or, in terms of Legendre normal forms,

$$I^0_{1,1} = \frac{4}{15\pi}\sqrt{\frac{\delta}{r_2}}\,\{2(3+\xi^2)E - (1+\xi)(3+\xi)F\} \tag{108.10}$$

if the eclipse is partial, and

$$I^0_{1,1} = \frac{8}{15\pi\kappa}\sqrt{\frac{\delta}{r^2}}\,\{(3+\xi^2)E - \xi(1+\xi)F\} \tag{108.11}$$

if it is annular.

22. An inspection of equations of preceding paragraphs shows that the functions $\mathfrak{J}^m_{\beta,\gamma}$ as well as $I^m_{\beta,\gamma}$ turn out to be also dimensionless quantities which can be made to depend upon two independent variables—say k and p—and tabulated in terms of such parameters similarly as the associated α-functions. Concerning the particular values which these functions may assume, it is evident that all boundary corrections should vanish at the moment of the first contact and—if $r_1 < r_2$—also at the beginning of the totality. In this latter case we thus again have

$$\mathfrak{J}^m_{\beta,\gamma}(k, \pm 1) = I^m_{\beta,\gamma}(k, \pm 1) = 0 \tag{109}$$

for any value of the subscripts and of m. If, however, $r_1 > r_2$, the boundary corrections do not vanish and remain finite [26] at the moment of internal tangency. Finally, at the moment of central eclipse

[26] If $r_1 > r_2$, the modulus of elliptical integrals in expressions with β and γ odd approaches unity at the moment of internal tangency, and hence $F\left(\frac{\pi}{2}, \kappa\right) \to \infty$. Since, however (*cf.* Whittaker and Watson, *op. cit.*, sec. 22.737),

$$\lim_{\kappa\to 1}\left\{F\left(\frac{\pi}{2}, \kappa\right) - \log\frac{4}{\sqrt{1-\kappa^2}}\right\} = 0,$$

it follows that

$$\lim_{\kappa\to 1}(1+\xi)F\left(\frac{\pi}{2}, \kappa\right) = \lim_{\kappa\to 1} 2(1-\kappa^2)F\left(\frac{\pi}{2}, \kappa\right) = 0$$

and the singularity is therefore only an apparent one.

($\delta = 0$), the only boundary corrections with non-vanishing coefficients are again those of zero order which reduce to

$$\pi\mathfrak{J}^0{}_{\beta,\gamma}(k, -1/k) = B(\tfrac{1}{2}, \tfrac{1}{2}\beta + 1)k^{\beta+3}(1 - k^2)^{\gamma/2} \tag{110}$$

and

$$\pi I^0{}_{\beta,\gamma}(k, -1/k) = B(\tfrac{1}{2}, \tfrac{1}{2}\beta + 1)k^{-\gamma}(1 - k^2)^{\gamma/2}. \tag{111}$$

Numerically, both the $\mathfrak{J}^m{}_{\beta,\gamma}$'s and $I^m{}_{\beta,\gamma}$'s diminish with increasing subscripts, and very rapidly with increasing superscript—much more rapidly than the respective associated α-functions. None of the terms entering in equations defining the theoretical light curves requires any additional integrations and hence the solution of our problem is complete.

VII. Discussion

23. In preceding sections formulae were set up which express the changes of light exhibited by close binary systems between minima as well as during eclipses, taking account of effects invoked by the second, third, and fourth spherical harmonic deformations. The results we found are by no means simple, and are likely to remain of little practical use unless the associated α-functions as well as functions expressing the boundary corrections are available in tabular forms. But once this has been done, theoretical light curves can be computed rigorously from a given set of parameters and compared with observations with minimum of difficulty.

The primary task which a computer of orbits of eclipsing binaries has to cope with is therefore the derivation of parameters specifying a light curve. In case of systems with well-separated components these can be obtained by a straightforward application of elementary methods to an observed light curve as it stands. If, however, the components get closer and the continuous changes of light between minima (due to the ellipticity and reflection effects) become too conspicuous to be neglected, Russell [27] proposed to "rectify" a light curve before analyzing it for orbital elements—that is, to apply empirically to the whole light curve certain harmonic terms required to render the light between minima constant. It is then assumed that such a rectified curve is equivalent to one that would be produced by the eclipse of two spherical stars, and to which the elementary methods of solution can be applied.

Let us investigate, in the light of the preceding results, the extent to which this assumption can hold true. If we generalize equations (88) for any value of p between ± 1 by putting

$$\alpha^{2\mu}_{2\nu}(k, p) = \frac{\nu!\Gamma(\mu + \frac{1}{2})}{\sqrt{\pi}(\mu + \nu + 1)!}\alpha_0{}^0(k, p) + A^{2\mu}_{2\nu}(k, p), \tag{112.0}$$

and

$$\alpha^{2\mu}_{2\nu-1}(k, p) = \frac{3\Gamma(\mu + \frac{1}{2})\Gamma(\nu + \frac{1}{2})}{2\sqrt{\pi}\Gamma(\mu + \nu + \frac{3}{2})}\alpha_1{}^0(k, p) + A^{2\mu}_{2\nu-1}(k, p), \tag{112.1}$$

then, to the second-order harmonic distortion, the "uniform" loss of light at any moment during eclipse assumes the form

$$\begin{aligned}\Delta\mathfrak{L}^U = \{1 + [2 + \tfrac{1}{4}\Omega_2{}^U][\tfrac{1}{3}v_1{}^{(2)}P_2(n_0) - w_1{}^{(2)}P_2(l_0)]\}\{\alpha_0{}^0 + [v_1{}^{(2)}n_0n_2 \\ - 3w_1{}^{(2)}l_0l_2][\alpha^1{}_{-1} + \Omega_2{}^U\alpha_1{}^1] + \tfrac{1}{2}\Omega_2{}^U[v_1{}^{(2)}(n_0{}^2 - n_1{}^2) - 3w_1{}^{(2)}l_0{}^2]A_2{}^0 \\ + \tfrac{1}{2}\Omega_2{}^U[v_1{}^{(2)}(n_2{}^2 - n_1{}^2) - 3w_1{}^{(2)}l_2{}^2]A_0{}^2 + \text{boundary corrections}\};\end{aligned} \tag{113.0}$$

while if the eclipsed star is completely darkened at limb,

$$\begin{aligned}\Delta\mathfrak{L}^D = \tfrac{3}{2}\{1 + \tfrac{4}{5}[5 + \tfrac{1}{2}\Omega_2{}^D][\tfrac{1}{3}v_1{}^{(2)}P_2(n_0) - w_1{}^{(2)}P_2(l_0)]\}\{\alpha_1{}^0 + [v_1{}^{(2)}n_0n_2 \\ - 3w_1{}^{(2)}l_0l_2][2\alpha_0{}^1 + \Omega_2{}^D\alpha_2{}^1] + \tfrac{1}{2}\Omega_2{}^D[v_1{}^{(2)}(n_0{}^2 - n_1{}^2) - 3w_1{}^{(2)}l_0{}^2]A_3{}^0 \\ + \tfrac{1}{2}\Omega_2{}^D[v_1{}^{(2)}(n_2{}^2 - n_1{}^2) - 3w_1{}^{(2)}l_2{}^2]A_1{}^2 + \text{boundary corrections}\}.\end{aligned} \tag{113.1}$$

Contributions to the light changes arising from the third and fourth harmonic tidal distortions can be rewritten similarly.

[27] *Ap.J.*, **36**, 54, 1912.

If we compare equation (113.0) or (113.1) with (73), we notice that in either case terms in the first curly brackets on the right-hand sides of (113.0) or (113.1) are identical with those invoking the variation of light between eclipses and can presumably be removed by rectification. The remaining terms, in second curly brackets, then represent the rectified theoretical light curve within minima. This, in addition to the fundamental modes $\alpha_0{}^0$ or $\alpha_1{}^0$, contains a number of terms of the first order which—however small—are of the same order of magnitude as those responsible for the variation of light between minima. The rectification therefore removes only a part of terms arising from the distortion. It can render a light curve simpler, but can by no means free it from all effects of distortion. Thus the vital assumption underlying the process of rectification—namely that a rectified light curve corresponds to one produced by the eclipse of two spherical stars—is not fulfilled to a higher approximation.

24. In view of the applications, it is desirable to pick up from (113) terms which are likely to be most important in practical cases. As the reader may easily visualize, an essential condition for the occurrence of eclipses is that the direction cosines l_2 and n_0 are of the same order of magnitude as the fractional radii of both components. Hence, as far as effects of the order of second harmonic distortions are concerned, quantities of the order of $l_2 w^{(2)}$ or $n_0 v^{(2)}$ are clearly negligible. The matters can be further simplified somewhat if we modify our definition of standard radius of a distorted star. So far (cf. sec. II) the standard radius has been defined as one of a sphere equal in volume to the distorted ellipsoid. This quantity does not, however, follow directly from preliminary orbital solution. The conventional "radii" of the components resulting from such a solution are very approximately equal to the diametral semi-axes of the actual ellipsoids.[28] If $a > b > c$ are the fractional semi-axes of an ellipsoidal star, then r, radius of a sphere of equal volume, is evidently $\sqrt[3]{abc}$; while the diametral semi-axis b, to the second-order harmonic distortion, becomes

$$b = r(1 - \tfrac{1}{6}v^{(2)} + \tfrac{1}{2}w^{(2)} + \cdots). \tag{114}$$

Let us eliminate by means of this equation r from (19) and subsequent equations, and adopt b as the standard radius of a distorted star. If we now abbreviate

$$\bar{v}^{(2)}_{1,2} = \left(1 + \frac{m_{2,1}}{m_{1,2}}\right)\Delta_2 b^3_{1,2}, \qquad \bar{w}^{(j)}_{1,2} = \frac{m_{2,1}}{m_{1,2}}\Delta j b^{j+1}_{1,2}, \qquad j = 2, 3, 4, \tag{10.1}$$

where the subscripts 1, 2 will refer to the primary (eclipsed) and secondary (eclipsing) component respectively, and make use of the algebraic identities

$$\mathfrak{I}^0_{1,0} + 2A_2{}^0 + 2A_0{}^2 = (r_2/r_1)^2 I^0_{1,2},$$

$$\mathfrak{I}^0_{-1,2} - 2A_2{}^0 = 0,$$

then by combining the boundary corrections due to the distortion of both components we find that the "uniform" rectified theoretical light curve reduces to

$$\Delta\mathfrak{V}^U_{\text{rec}} = \alpha_0{}^0 + \{\bar{v}_1{}^{(2)} - \bar{v}_2{}^{(2)}\}(r_2/r_1)^2 I^0_{1,0} - \{2 + \tfrac{1}{2}\Omega_2{}^U\}\{\bar{v}_1{}^{(2)}[A_2{}^0 + A_0{}^2] + 3\bar{w}_1{}^{(2)}A_2{}^0\} + \cdots. \tag{115}$$

To this order of accuracy the rectified light curve does not involve the direction cosines l or n, and the light changes are therefore independent of the scale of each particular system.

Of the two first-order terms on the right-hand side of (115), the term

$$\{\bar{v}_1{}^{(2)} - \bar{v}_2{}^{(2)}\}(r_2/r_1)^2 I^0_{1,0} \tag{116}$$

is purely geometrical and accounts for the difference in polar flattening of both components; while

$$\{2 + \tfrac{1}{2}\Omega_2{}^U\}\{\bar{v}_1{}^{(2)}[A_2{}^0 + A_0{}^2] + 3\bar{w}_1{}^{(2)}A_2{}^0\} \tag{117}$$

arises in connection with gravity-darkening. The former term consists of a difference of small quantities, and unless both components are very unequal in form it is likely to be minor. The latter consists, however, of a sum of small quantities and is therefore likely to be the dominant one. To this order of accuracy, the rectified light curve does not depend upon the tidal distortion of the

[28] *I.e.*, to the radii of their diametral cross-sections which are circles for tidally distorted stars.

secondary component.[29] This is easy to understand; for the tides do not affect the diametral cross-section (exposed to us within minima) of the distorted stars. Tidal distortion of the eclipsed star enters into (115) only indirectly, through the gravity-darkening.

Were the gravity effect non-existent, it would follow from equation (68) that $\Omega_2^U = -4$, in which case (117) vanishes; and so does (116) if $\bar{v}_1 = \bar{v}_2$. Thus, for uniformly bright stars, the process of rectification would apply rigorously if both components were prolate spheroids or similar three-axial ellipsoids. These facts have already been pointed out by Russell.[27] On the other hand, if we take the coefficient of gravity-darkening appropriate for integrated radiation, $\Omega_2^U = 0$, and equations (113) or (115) also undergo considerable reduction. There is, however, no special likelihood that Ω_2^U should vanish, and it is physically impossible that it could reach -4. In practical cases Ω_2^U will generally be neither zero nor an integer, and may be positive or negative. Equation (115) is then to be used as it stands.

In the case of limb-darkening, equation (113.1) admits of an analogous simplification, but the functions associated with the distortion are considerably more complicated. Fortunately, it does not appear necessary to investigate them at present to the same detail. For equation (17.0) can evidently be rewritten as

$$\Delta\mathfrak{L} = \Delta\mathfrak{L}^U + \frac{2u}{3-u}(\Delta\mathfrak{L}^D - \Delta\mathfrak{L}^U). \tag{17.1}$$

The "darkened" functions are therefore found to enter, not in their absolute values, but only through differences between the corresponding "uniform" and "darkened" functions. If the coefficient of darkening is small, then within the scheme of our approximation we may neglect the differences of small quantities and retain only the zero-order term

$$\frac{2u}{3-u}(\alpha^D - \alpha^U);$$

but if the darkening is complete or nearly so, the differences of small quantities may become noticeable and should be considered.[30]

25. Equations (113) or (115) hold good no matter whether the eclipse is partial or annular. In the latter case, however, (115) admits of considerable simplification. For in section VI we found that, during annular phase, $\alpha_0^0 = k^2$, $I^0_{1,0} = \frac{1}{2}$, and

$$A_0^2 = +k^2\{D^2 - \tfrac{1}{4}(1-k^2)\}, \tag{118.0}$$

$$A_2^0 = -k^2\{D^2 - \tfrac{1}{2}(1-k^2)\}. \tag{118.1}$$

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[29] As was already discussed by the writer in *Ap.J.*, **94**, 159, 1941 (sec. IV), the effect of tidal distortion of the eclipsing component, small as it is, tends to simulate increased limb darkening of the eclipsed star. The proof as given in section IV of that paper is, unfortunately, obscured by the fact that radius of a sphere of equal volume as the distorted ellipsoid was used as the standard radius of the secondary component. If, as above, the diametral semi-axis is adopted as standard radius instead, the radial-expansion terms $I^0_{-1,0}$ or $I^0_{-1,1}$ disappear from equations (9) or (10) of the paper referred to, while equation (9.1) of the same paper should be replaced by

$$k^2 f_2^U \doteq 3\bar{w}_2^{(2)}\delta^2 I^2_{-1,0} - \bar{v}_2^{(2)} I^0_{1,0}.$$

Neglecting differences of small quantities, the loss of light due to the eclipse of a spherical star becomes

$$\Delta\mathfrak{L} = \alpha^U + f_2^U + \frac{2u}{3-u}(\alpha^D - \alpha^U).$$

Near the inner contact and in advanced partial stages $I^2_{-1,0}$ and $I^0_{1,0}$ are positive and increasing functions of the geometrical depth; but so is the difference $\alpha^D - \alpha^U$. Hence the effect of the tidal term in equation (9.1) will evidently be to simulate increased limb darkening of the eclipsed star, while the rotational term will tend to do the opposite. These are the conclusions reached in the writer's previous paper. The validity of equivalence between distortion and darkening is, however, limited; and on the basis of more numerical computations it appears now dubious to the writer whether equations (28.0) and (28.2) of that paper or analogous relations can possess much practical meaning.

[30] Provided, of course, that the degree of limb-darkening is sufficiently well known; for otherwise the uncertainty of the zero-order term, caused by our incomplete knowledge of darkening, may introduce errors comparable with those arising from the neglect of differences of small quantities.

Hence $A_0^2 + A_2^0 = \frac{1}{4}k^2(1 - k^2)$, and the rectified uniform loss of light then takes the form

$$\Delta\mathfrak{L}^U = \alpha_0^0\{1 + \tfrac{1}{2}[\bar{v}_1^{(2)} - \bar{v}_2^{(2)}] - \tfrac{1}{2}(1 - k^2)(1 + \tfrac{1}{4}\Omega_2^U)[\bar{v}_1^{(2)} + 6w_1^{(2)}] + \tfrac{3}{2}(1 + \tfrac{1}{4}\Omega_2^U)w_1^{(2)}D^2 + \cdots\}. \quad (119)$$

Of the two terms in curly brackets on the right-hand side of equation (115) the first one, arising from the difference in polar flattenings of both components, becomes constant during annular phase, and so does the second term in so far as it depends on flattening of the eclipsed star. Thus, to the order of accuracy we are working, the only term varying with the geometrical depth is

$$+ \tfrac{3}{2}(1 + \tfrac{1}{4}\Omega_2^U)w_1^{(2)}D^2, \quad (120)$$

and arises from the gravity-darkening due to the tidal distortion of the eclipsed primary star. In the absence of ordinary limb-darkening the rectified loss of light should therefore reach its maximum at the moment of internal tangency, and decrease henceforth continuously with diminishing geometrical depth of the eclipse; a rectified light curve should be convex upwards during annular phases. This is easy to understand; for the distribution of brightness on the surface of a tidally distorted star, due to the gravity effect and viewed from the direction of its longest axis, is tantamount to limb-brightening.[31] If, however, the limb- and gravity-darkenings superpose, the situation becomes too complex to be analyzed in simple terms; and the increase of brightness from center to limb needs no longer to take place. The rectified light during annular phases may then remain sensibly constant, or may actually decrease with decreasing geometrical depth, as it would do if the eclipsed star were darkened at limb but the gravity-darkening were absent.

26. The application of the preceding results to practical cases can best proceed by the "method of false position." [32] After having performed an empirical rectification we neglect, at first, the remaining effects of distortion and solve our light curve by elementary methods. With the aid of rough elements thus obtained we compute the true theoretical light curve, including the effects of distortion, and compare with observations. The simplified forms (115) or (119) can preferably be used for the second approximation. Let O be the observed curve, and C the computed one. Construct now a new curve which is at every point as far below O as C is above it. From this curve, by the same method of solution, derive new elements and from these a new computed curve C'. The latter should be much closer to O than C was. Repetition of this process, if necessary, should result in a good fit. There is no general guarantee that such approximations will converge in every practical case; yet if they fail, there is but little hope that the far more laborious adjustment by least squares would improve the results. Only after a really good fit has thus been obtained is it justifiable to attempt the latter—remembering that the test of a good preliminary job is that the least-squares corrections should be of the same order of magnitude as their probable errors.

The procedure described above is not likely to be unduly laborious, but is admittedly much more involved than the simple form of solution applicable if the effects of distortion are neglected. As long as both components can be regarded as circular disks with uniform or radially symmetrical distribution of brightness the orbital problem is simple; and to this extent it was exhaustively dealt with in the classical memoirs by Russell and Shapley [33] thirty years ago. At that time the photoelectric photometry was still in its infancy; and errors of normal magnitudes constituting the best light curves then available were as a rule measured in hundredths of a magnitude. In the three decades which elapsed since that time, however, the accuracy of photometric measurements has been increased about ten times; and precise light curves of several close binaries, produced in recent years, have exceeded very definitely our ability of interpreting them; the simplified model underlying Russell's method proved no longer adequate.

The aim of the present paper has been to start from a model corresponding more closely to reality, and to extend the background of our interpretation of light curves as far as it proves practicable at

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[31] *Cf.* Russell, *Ap.J.*, **90**, 641, 1939 (p. 660).

[32] The writer owes this suggestion to Professor Russell. (*Cf.* a forthcoming paper by Professor Russell in the *Astrophysical Journal*.)

[33] Russell, *Ap.J.*, **35**, 315, and **36**, 54, 1912; Russell and Shapley, *Ap.J.*, **36**, 239, 385, 1912.

the present state of research. And it is believed that, unless a system is so close that its components are nearly in contact, our results should enable one to carry out an analysis of light curves, based on a physically sound and consistent model, up to quantities of the order of one or two thousandths of a magnitude. The situation arising thus in orbital analysis of eclipsing binaries may be compared with that experienced in the early days of celestial mechanics, when the Keplerian ellipses—an analogon to the simplified form of problem treated by Russell and Shapley—proved inadequate to represent the orbits of bodies in our solar system with sufficient accuracy, and recourse had ultimately to be made to the theory of general perturbations.

In conclusion, the writer takes pleasure in expressing his appreciation to Professor Henry Norris Russell for stimulating interest, and to Dr. Harlow Shapley for constant generous encouragement to which the present paper owes a great deal.

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